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APPLICATION OF WAVELETS ANALYSIS TO MAGNETIC FIELD SOURCE SEARCHING

K.NAKAJIMA, S.HAYANO and Y.SAITO
College of Engineering, Hosei University, Kajino, Koganei, Tokyo 184, Japan

ABSTRACT

Discrete wavelets transform is widely used for the wave and image analyses. Particularly, data compression ability is useful tool for the image data analysis. On the other side, discrete wavelets analysis can be applied to the linear systems, because wavelets transform is one of the linear transformations in linear space.

In the present paper, we propose an inverse approach employing the discrete wavelets transform. An optimal sensor position can be decided by means of the correlative analysis for the wavelets solution of ill posed system equations. Our method was applied to an estimation of the high frequency current distributions on a film conductor. Intensive experimental verification shows the validity of our method.

KEYWORDS

Wavelets transform, optimal sensor layout, ill posed problem, skin effect.

INTRODUCTION

Continuous wavelets transform based on the concept of Fourier analysis has been recently proposed. However, it is difficult to perform an inverse transformation of the continuous wavelets, because their base functions are not orthogonal. On the other side, base functions of the discrete wavelets transform are orthogonal. This discrete wavelets transform is simply linear transformation and has been usefully applied to the data compression of the wave and image data [1].

With the developments of modern superconducting quantum interference device (SQUID), it becomes possible to measure the magnetic fields around a human brain accompanying with brain operation. To clarify the human brain operation, searching for the magnetic field source distributions from the locally measured fields is of paramount importance. This means that it is essential to solve an ill posed linear system of equations. Various numerical methods have been proposed solving for the ill posed systems. In biomagnetic fields, the least squares and minimum norm methods are widely used [2-4]. The former is applied to finding the most dominant single field source, i.e. current dipole, and the latter is used to identify the field source distributions. Further, new techniques have been proposed in order to evaluate the reliable and unique solutions [5-6].

Previously, we have proposed a wavelets approach solving for the ill posed linear system of equations [7-8]. Key idea of this approach is that the two dimensional wavelets transform is applied to the system matrix and collecting the most dominant elements on the system matrix yields an approximate inverse matrix in the wavelets spectrum domain. Inverse wavelets transform of this approximate inverse matrix gives an approximate inverse matrix in the original domain. Thus, it is possible to obtain the approximate solution of ill posed linear systems. However, this wavelets approach has a serious difficulty that a set of dominant elements in wavelet spectrum domain does not always have an inverse matrix.

To overcome this difficulty, this paper proposes a novel method that assumes a positive definite part wavelets spectrum matrix. After recovering the original size spectrum matrix, inverse wavelets transform of the spectrum matrix yields a reasonable system matrix. Correlative analysis between this reasonable and practical system matrices changing the field measuring conditions gives the best sensor layout for the wavelets solution. As an example, we have applied this new strategy to an estimation of the high frequency current distributions on a film conductor. As a result, it is revealed that the computed values correspond well to the experimental ones.

BASIS EQUATIONS

A relationship between magnetic field H and current density J is given by

$$H = \int_V G\, J dv \quad , \tag{1}$$

where G' is space derivative of Green function and v is the volume containing the current density J.

Now, assuming the sufficiently small volume Δv, Eq.(1) can be discretized as

$$\mathbf{X} = D\mathbf{Y} \quad , \tag{2}$$

where the vectors X, Y and system matrix D are respectively

$$\begin{aligned} \mathbf{X} &= \begin{bmatrix} H_1 & H_2 & . & H_n \end{bmatrix}^T, \\ \mathbf{Y} &= \begin{bmatrix} \Delta v J_1 & \Delta v J_2 & . & \Delta v J_m \end{bmatrix}^T, \\ &= \begin{bmatrix} i_1 & i_2 & . & i_m \end{bmatrix}^T, \quad i_j = \Delta v J_j, j = 1,2,.,m, \\ D &= \begin{bmatrix} G_{11} & G_{12} & . & G_{1m} \\ G_{21} & G_{22} & . & G_{2m} \\ . & . & . & . \\ G_{n1} & G_{n2} & . & G_{nm} \end{bmatrix}, \end{aligned} \tag{3}$$

where n is the number of field measured points, m is the number of subdivisions $\triangle v$, and a condition $n<<m$ is held.

APPROXIMATE SOLUTION OF THE ILL POSED LINEAR SYSTEMS

WAVELETS TRANSFORM

Equation (4) is the wavelets transfromed system equation of (2).

$$\mathbf{X}' = D'\mathbf{Y}', \qquad \mathbf{X}' = W_n\mathbf{X}, \quad \mathbf{Y}' = W_m\mathbf{Y}, \quad D' = W_n D W_m^T, \tag{4}$$

where D' and D are the transformed(spectrum) matrix and original matrix with order n by m, respectively. W_n and W_m are the wavelets transform matrices with order n by n and m by m, respectively.

APPROXIMATE SOLUTION

The transformed system matrix D' is a still singular matrix, because the original system matrix D is a singular matrix. According to the nature of wavelets transform, D' is approximated by a part matrix d which is composed of the top n' by n' square part in the wavelets spectrum matrix. By means of a reasonable selection of n', it is possible to obtain a positive definite matrix d. Then, the approximate inverse matrix is given by

$$D'^{-1}_{Appro} = d^{-1}. \tag{5}$$

Similarly, vector X' is approximated by X'_{Appro} from the first to the n'th elements. The spectrum solution vector Y' is given by

$$\mathbf{Y}'_{Appro} = D'^{-1}_{Appro}\mathbf{X}'_{Appro}. \tag{6}$$

After adding the $(m\text{-}n')$th zero elements to Y'_{Appro}, we have an approximate solution vector Y_{Appro} in the original space, viz.,

$$\mathbf{Y}''_{Appro} = \left[\mathbf{Y}'_{Appro}, 0, \ldots, 0\right]^T, \qquad \mathbf{Y}_{Appro} = W_m{}^T\mathbf{Y}''_{Appro}. \tag{7}$$

This is the basic principle of ill posed linear system solution strategy employing wavelets transform.

THE ESTIMATION OF THE HIGH FREQUENCY CURRENT DISTRIBUTIONS ON A FILM CONDUCTOR

MODEL

As shown in Fig.1, a film conductor is represented by one dimensional current array model having m conductors. Magnetic field distribution above this current array is measured at n locations. In Fig.1, L_f is the film width, $\triangle Y$ is the hight of the magnetic field measured position and L_m is the length of the measurement surface.

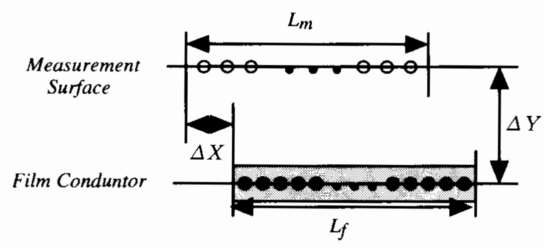


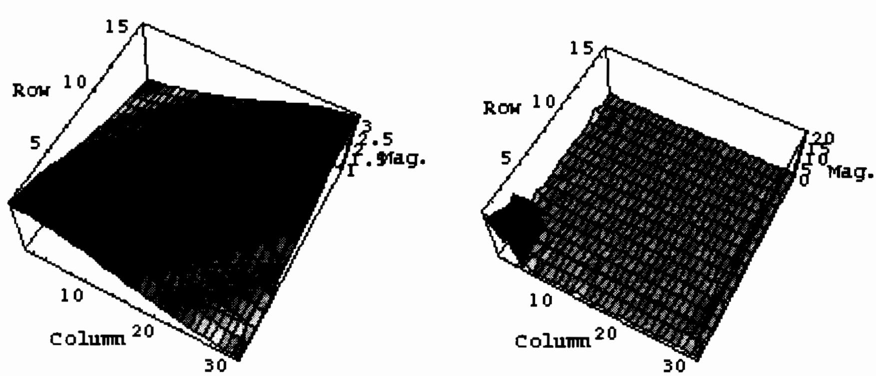
Fig.1 Modelling of current distributions on a film conductor

SIMULATION

Film width L_f , measurement region L_m , $\triangle Y$ and $\triangle X$ were set respectively at 8.0, 8.0, 5.0 and 0[cm]. Fig.2(a) shows the system matrix obtained by setting $n=16$ and $m=32$. The magnetic fields in the direction of parallel to the film surface were measured. Employing Daubechies 8 order base function, the system matrix D was transformed into spectrum matrix D' shown in Fig.2(b).

Taking only 8×8 elements from the left bottom corner of Fig.2(b) resulted in a well-conditioned matrix d. The condition used for deriving the matrix d is relative error of the Gaussian elimination processes for the matrix d inversion. In this case, the relative error was 6.0×10^{-9}[%] for the matrix d. Figures 3(a) and 3(b) show respectively the calculated value and the exact solution.

By comparison of the results in Figs.3(a) with 3(b), simple wavelets solution strategy yields a vibrating solution. This means that the accuracy of inverse matrix d^{-1} is not sufficient.



(a) (b)

Fig.2 Ill posed system matrix and its spectrum matrix, (a)System matrix D, (b)Wavelets spectrum matrix D'

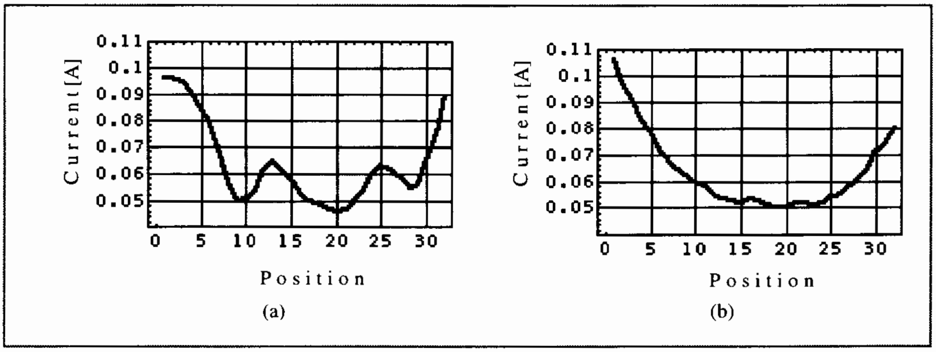
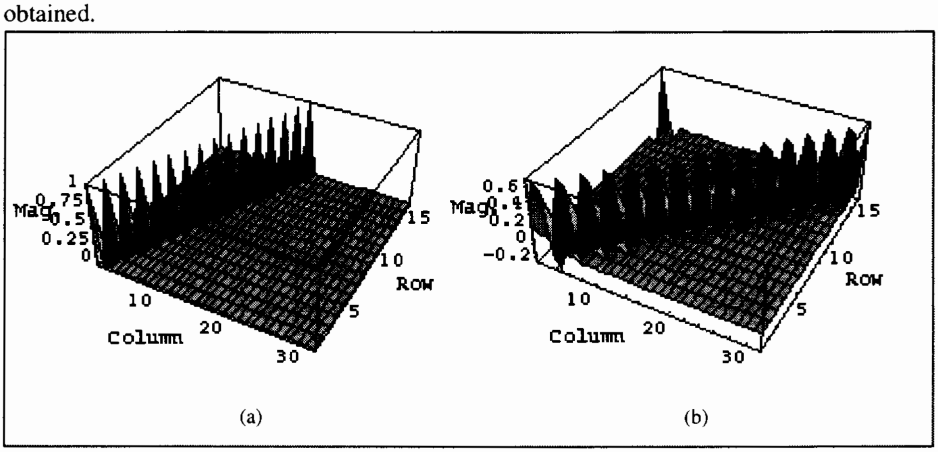


Fig.3 Comparison of approximate and exact solutions, (a)Wavelets solutions, (b)An exact solution

OPTIMAL SENSOR LAYOUT

To improve the accuracy of the approximate inverse matrix, we change the field measuring condition. Let us consider the wavelets transformed spectrum matrix D' shown in Fig.4(a). Obviouly, the part spectrum matrix d in the wavelets spectrum has assumed to be idendity matrix. Accordingly, a reasonable system matrix D can be derived by applying the inverse transform to this spectrum matrix D'. Fig.4(b) shows the reasonable system matrix using Dabechies 8 order base function. Fig.4(c) shows a result of correlative analysis between this reasonable and the practical system matrices changing sensor height. There is a peak in the correlative coefficients, apparantly. The height $\triangle Y$ to this peak represents an optimal sensor position. Fig.4(d) shows the practically optimized system matrix D. Fig.5(a) shows the wavelets spectrum D' of the practically optimized system matrix D using Daubechies 8 order base function. The relative error of the part spectrum matrix d taken only 8×8 elements from the left bottom corner of Fig.5(a) is 4.2×10^{-14}[%]. Fig.5(b) shows the computed current distributions.

Comparison of the results in Figs.5(b) with 3(b) reveals that well solutions could be obtained.



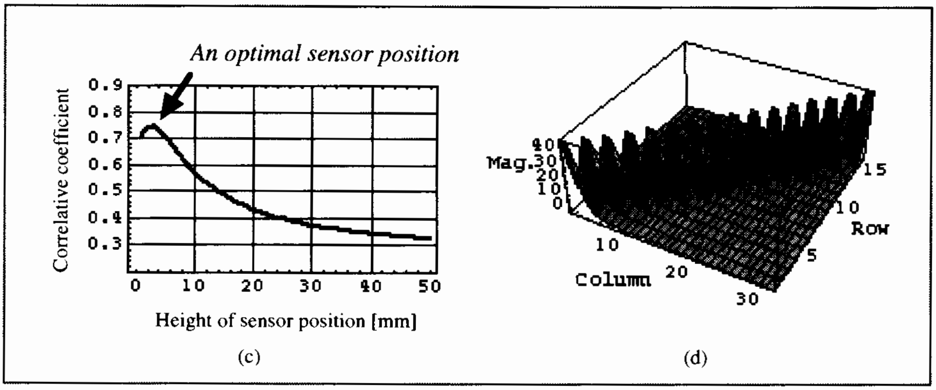


Fig.4 Optimal sensor layout and optimal system matrix, (a)Reasonable wavelets spectrum, (b)Reasonable system matrix, (c)Sensor position vs correlative coefficient, (d)Practically optimized system matrix

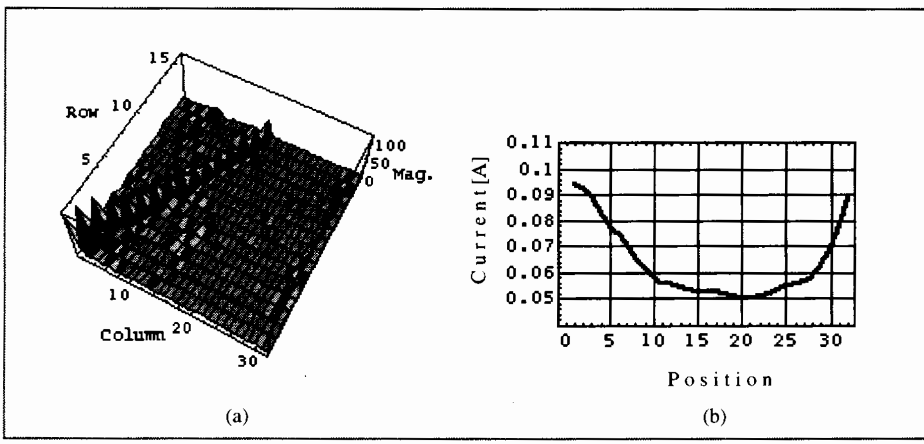


Fig.5 The wavelets spectrum and solution under the optimized sensor position, (a)Wavelets spectrum matrix D', (b)Optimal wavelets solution

EXPERIMENT

In order to measure the current in the film conductor, enameled wires in stead of the film conductor have been put parallel as shown in Fig.6. Thus, the each current in the enameled wires can be measured directly.

Film width L_f, measurement region L_m, $\triangle Y$ and $\triangle X$ were fixed respectively at 8.0, 8.0, 0.3 and 0 [cm]. The system matrix has been obtained from $n=16$ and $m=32$. Fig.7 shows the comparison of the computed and experimental current distributions. Obviouly, the solution obtained from the optimal sensor layout corresponds well to the experimental one. Changing the driving frequency demonstrates the difference of skin effect.

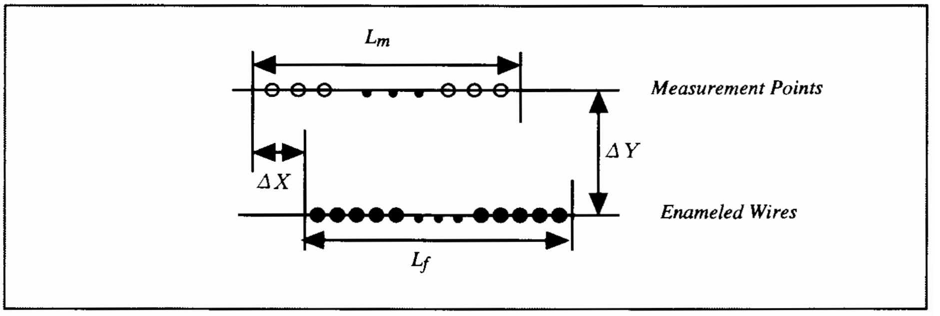


Fig.6 A schematic diagram of the experiment

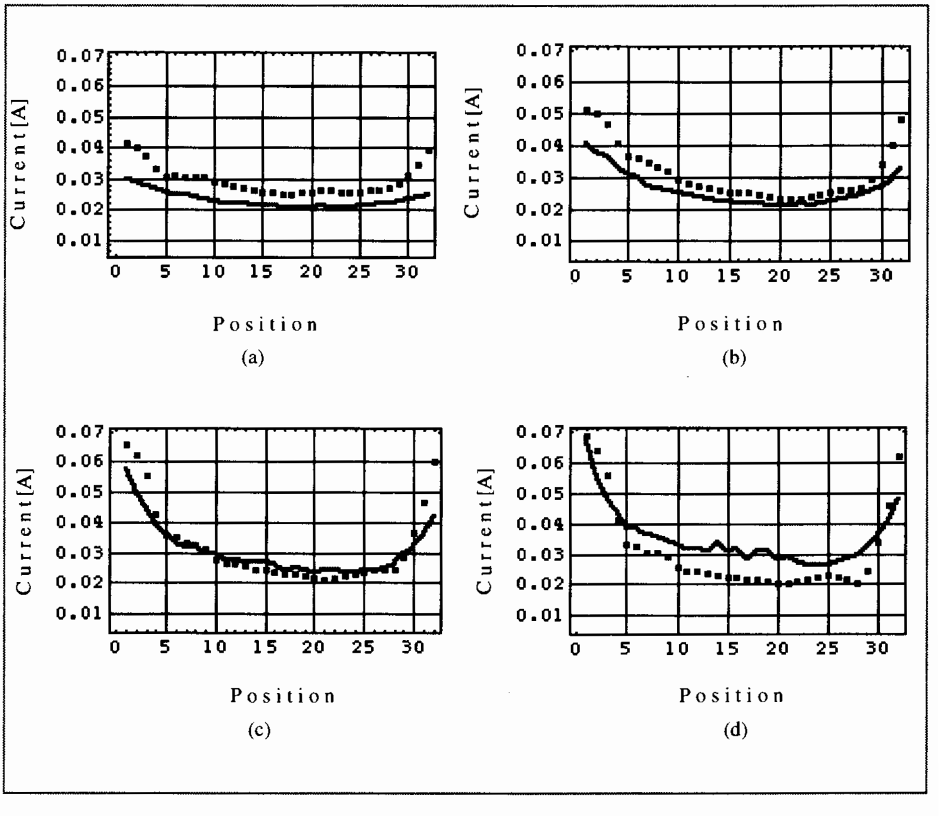


Fig.7 Comparison of the experimental and computed current distributions, Solid and dotted lines refer to the experimental and computed solutions, respectively, (a)50kHz, (b)100kHz, (c)200kHz, (d)300kHz

CONCLUTIONS

As shown above, we have proposed a new wavelets solution strategy for the ill posed linear systems. Application of our method to the estimation of current distributions in a film conductor has demonstrated the usefulness of our approach.

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