DISCRETE WAVELETS TRANSFORM ANALYSIS IN AXIAL VELOCITY DISTRIBUTION OF SPIRAL FLOW

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ABSTRACT
Time-frequency distributions of axial turbulence velocities of spiral pipe flow and typical turbulence flow have been clearly decomposed in a range from low frequency level (15.625 Hz) to high frequency level (500.0 Hz) by means of discrete wavelets transform. As a result, the lower frequency levels (under Level 4 = 1250 Hz) of the spiral flow are extremely lower as compared with those of the typical turbulence flow. Moreover, the spiral flow is dominated by Level 3 to be stabilized from the autocorrelation. The originality of this paper lies in applying discrete wavelets transform and its autocorrelation analysis to analyzing the spiral flow stable motion in time-frequency dimension.

KEYWORDS Discrete wavelets transform, Spiral flow, Frequency analysis, Turbulence velocity, Fluctuation level, Autocorrelation

INTRODUCTION
Spiral flow is a swirling flow with large free vortex region, high concentration to the axis and high stability [Horii et al. 1990]. From the high stability characteristics, the spiral flow is useful for industrial applications such as optical cord installation in a small diameter pipeline with bends [Horii et al. 1991] and high performance pneumatic transportation without particles touching pipe inner wall [Takei et al. 1997]. The solids in the two-phase spiral pipe flow acquire stably their position in a pipeline without large vibration. The motivation behind this work is to clarify the mechanism of the high stability in order to improve the spiral flow system. Time-frequency analysis is a suitable method to analyze the stability. Recently, wavelets transform has been popular for time-frequency analysis instead of Fourier transform in mechanical engineering fields. The merits of the wavelets analysis is to be able to analyze the frequency not to eraser the time information.

Wavenets transform [Moret 1989] is roughly classified with two types, which are continuous wavelets transform and discrete wavelets transform. The continuous wavelets transform has been generally used for time frequency analysis in vibration wave. For example, self-similarity of the inner structure of the jet [Everson 1990], the breakdown of a large eddy and the successive branching of a large eddy structure in a plane jet [Li 1995], decomposition of Reynolds stress in a jet [Gordienko 1995], and the multiple acoustic modes and the shear layer instability [Walker 1995] were investigated. However, most of the researchers on the time-frequency analysis carried out the continuous wavelets transform. On the other hand, the discrete wavelet transform has been mainly used for picture image processing. The analysis enables to decompose and to compose of picture image data quantitatively because of the orthonormal transform. Saito applied this idea to analyzing the electromagnetic wave [Saito 1996]. The originality of this paper lies in applying discrete wavelets transform and autocorrelation to each frequency level to analyzing the spiral flow stable motion. In this paper, as a first step to clarify the stability, time-frequency distribution of axial turbulence velocity of spiral pipe flow is decomposed from low frequency level (15.625 Hz) to high frequency level (500.0 Hz) by discrete wavelets transform and its autocorrelation. It is recognized which level is dominant to stabilize the spiral flow.

THEORY OF DISCRETE WAVELETS TRANSFORM
Basic Concept Using Simple Base Function
Basic concept of discrete wavelets transform is described using matrix expression instead of integral expression. One dimensional input data matrix with four elements X and an analyzing wavelets matrix of Haar base function W are used to simplify the expression. For example, the input data matrix X is discrete velocity data with time. The wavelets transform matrix S that indicates wavelets spectrum is expressed by

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where \( C^2 = \text{cof} \). The first line in Eq. (6) is called scaling coefficients and second line is called wavelets coefficients. Forth Daubechies function (N=4) has four coefficients in a line. The first line shows a transform to obtain a mean value with weights of \( c_0, c_1 \) and \( c_2 \) on the input data. The second line shows a transform to obtain a difference value with weights of \( c_0, c_2 \) and \( c_3 \) on the input data. The third line shows a transform to translate the first line by two steps. The fourth line is a transform to do the second line by two steps. Eqs. (7) and (8) show the transform values are zero when the input data are constant or are simply increased. To explain easily the process to acquire the analyzing wavelets matrix \( W \) from \( C \), the matrix \( X \) is assumed as one dimensional 16 elements,
\[
X = [x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}]
\]
(10)
Where, \( C_{16} \times 16 \) matrix of \( C \). The element \( x \) indicates the mean value and the element \( c \) indicates the difference value. The elements in the matrix \( X \) are replaced by a matrix \( P_{16} \)
\[
P_{16}X = C_{16}X = [x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}]
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\]
(10)
\[ X = [W_{S0}^{0}] \mathbf{S} \]  
\[ [W_{S0}^{0}] = ([P_{s_0}^{0}, C_{s_0}^{0}], [P_{s_0}^{0}, C_{s_0}^{0}], [P_{s_0}^{0}, C_{s_0}^{0}])^T \]  
\[ [C_{s_0}^{0}, P_{s_0}^{0}, C_{s_0}^{0}], [P_{s_0}^{0}, C_{s_0}^{0}], [P_{s_0}^{0}, C_{s_0}^{0}])^T \]  
\[ S_0 = [S_0, \ldots, S_0] \]  
\[ S_0 = [0 \ldots 0, D_0, 0 \ldots 0, 0 \ldots 0] \]  
\[ S_0 = [0 \ldots 0, D_0, D_1, D_2, D_3] \]  
\[ S_0 = [0 \ldots 0, D_0, D_1, D_2, D_3, D_4] \]  
\[ \text{In the case of sixteen input data and fourth Doulbechies,} \]  
\[ \text{multiresolution is from Level 0 to Level 3. In general,} \]  
\[ \text{in the case that input data is 2d and Doulbechies function is 4th} \]  
\[ \text{(N=4), the algorithm to obtain levels is shown in Fig. 1. The} \]  
\[ \text{final wavelets spectrum is obtained after the wavelet transform} \]  
\[ \text{in Eq. (14) continues until the number of final summation} \]  
\[ \text{elements is less than k.} \]  

**EXPERIMENTS**

**Nozzle to Produce Spiral Flow**

The nozzle to produce the spiral flow is designed with an annular slit connecting to a conical cylinder as shown in Fig. 2. The pressurized air is forced through the sides of the device into the buffer area, and then through the annular slit into a vertical pipe entrance. The suction force is generated at the back of the nozzle by Coanda effect. The annular flow, passing through the conical cylinder, develops a spiral structure with a steeper axial velocity and an azimuthal velocity distributions, even if it is not applied tangentially. Vaporized water as a tracer of LDV are sucked into the nozzle from the back of the nozzle. An ejector is used to generate the typical turbulence flow.

**Fig. 2 Spiral flow nozzle**

**Experimental Equipment, Method & Conditions**

The experimental equipment consisted of a vertical acrylic pipe, the nozzle to produce the spiral flow and an air compressor as shown in Fig. 3. The inside diameter of the vertical pipe was 41.0 mm. A LDV probe is set up at the side of the vertical pipe at 1.0 m from the air supply part to measure the axial velocity at the center of the pipe. The He-Ne Laser power of LDV was 10 mW, and the probe pick up the reflected wave from the tracer. The air flow rate was 1.98 X 10^{-3} m^3/s. The mean velocity of the air flow in the vertical pipe calculated from the flow rate was 1.50 m/s. The Reynolds number calculated from the mean velocity was about 4200.

The reflected wave pass through a timer unit connecting to LDV probe for 1 ms (1,000 Hz) pick-up interval. The signals of the reflected wave were counted for about 5 seconds in a counter system connecting to the timer unit. The discrete sampling velocity data were 1024 (4096). The counter system has high pass and low pass filters that reduce signals under 0.625 m/s and over 6.25 m/s as noise. The pick up point is one point where is the center of the pipe as a first step study. The time mean velocities and turbulence levels of the spiral flow and typical turbulence flow are compared.

**Fig. 3 Experimental equipment**

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Experimental Results

The velocities of the spiral flow and typical turbulence flow at the center of the pipe are obtained with LDV. The turbulence level is defined as,

$$\nu^* = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{v_i - \nu_{\text{mean}}}{\nu_{\text{mean}}} \right)$$

(22)

Where, \( n \) is the sampling velocity number, \( \nu_{\text{mean}} \) is the time mean velocity and \( v_i \) is a velocity in a time. The time-mean velocity and the turbulence level are shown in Table 1. From this table, the time mean velocity of the spiral flow is higher than that of typical turbulence flow by about 9% even though the air flow rate is the same [Hori 1990]. That is because the axial velocity of the spiral flow is steeper than that of the typical turbulence flow. Also, the turbulence level of the spiral flow is much lower than the typical turbulence flow by about 10%. It means the spiral flow is a stable flow in an axial direction. The normalized axial turbulence velocities \( \nu^* = (v_i - \nu_{\text{mean}}) / \nu_{\text{mean}} \) are shown in Figs. 4 and 5. These figures are analyzed in the next section.

<table>
<thead>
<tr>
<th>Table 1 Time-mean velocity and turbulence Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time-mean velocity ( \nu_{\text{mean}} )</td>
</tr>
<tr>
<td>Spiral Flow</td>
</tr>
<tr>
<td>Typical Turbulent Flow</td>
</tr>
</tbody>
</table>

ANALYSIS AND DISCUSSION

Analysis Method

The normalized axial turbulence velocities in Figs. 4 and 5 are analyzed by discrete wavelet transform and its autocorrelation. This wavelet analysis consists of three steps. Firstly, the 1024 \( (2^{10}) \) sampling data of the axial turbulence velocities are put into the matrix \( X \) in Eq. (9). The matrix \( X \) is transformed to the wavelet spectrum \( S \) in Eq. (14). Next, the multiresolution analysis is carried out, that is, each part of the spectrum is inversely transformed to multiresolution levels by means of the discrete inverse wavelets transform in Eq. (20). Finally, to recognize which level is dominant for the spiral flow stability, autocorrelation of each level is obtained. Twentieth Daubechies function is used as an analyzing wavelets function. Twentieth Daubechies function has twenty coefficients from \( c_{10} \) to \( c_{20} \). In the first line in Eq. (6), twenty coefficients from \( c_{10} \) to \( c_{20} \) in the second line in Eq. (6). In the case of twentieth Daubechies function and 1024 \( (2^{10}) \) input data, the multiresolution classifies to seven levels as shown in Eq. (23).

$$X = [W^{09}]S = [W^{09}]S_{10} + [W^{09}]S_{10} + [W^{09}]S_{10} + [W^{09}]S_{10} + [W^{09}]S_{10} + [W^{09}]S_{10} + [W^{09}]S_{10}$$

(23)

\( W^{09} \) indicates the five times operation to obtain Daubechies matrix from a matrix \( C \) in Eq. (6). The coefficients of twentieth Daubechies function are shown in Fig. 6. \( x \) axis shows the coefficients from \( c_{10} \) to \( c_{20} \) in the second line of \( C \) matrix in Eq. (6).

Therefore, 1 in \( x \) axis indicates \( c_{10} \), 2 in \( x \) axis indicates \( c_{10} \), and 20 indicates \( c_{20} \) in Fig. 6.

Fig. 6 Coefficients of twentieth Daubechies function

Turbulence Level on Each Frequency Level

To clarify the difference between waveforms transform and Fourier transform, the axial turbulence level on each frequency level is defined in Eq. (24) calculated before indicating the wavelet analysis.

$$\nu^* = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{v_i - \nu_{\text{mean}}}{\nu_{\text{mean}}} \right)$$

(24)

Where, \( m \) is a frequency level, \( \nu_{\text{mean}} \) is a time mean turbulence velocity and \( v_i \) is a turbulence velocity in a time on each wavelet level. The \( \nu^* \) indicates a kind of normalized turbulence level obtained by Fourier transform. The mean turbulence velocity on each frequency level \( \nu_{\text{mean}} \) is not zero. The turbulence level on each frequency level is shown in Fig. 7. Level 0 is not shown. From this figure, the turbulence levels of the spiral flow on all levels are lower than those of typical turbulence flow. Mainly, the level from Level 1 (15.625 Hz) to Level 4 (125.0 Hz) are remarkably different. Both turbulence levels have peaks at Level 3 (62.5 Hz). The relation between the frequency level and the wave number is shown in Table 2. If Kolmogorov wave number \( k_0 \) is assumed to be 10<sup>2</sup> order, the wave number range normalized with \( k_0 \) is from \( 10^{-2} \) to \( 10^{-1} \). It means that range includes the energy contain range and the inertial range. The position of the peaks are reasonable from Kolmogorov theory.
Waves analysis results & discussion

Transforming inversely each level of the wavelets spectrum indicates multiresolution as shown in Eq. 20. Fig. 8(A) shows the multiresolution of the spiral flow, and Fig. 8(B) shows the multiresolution of the typical turbulence flow in three dimension display. From Fig. 8, it is recognized that time and frequency level is simultaneously analyzed. To clarify the each frequency level, Fig. 8 is displayed in two dimension as shown in Fig. 9. From this multiresolution, the spectrum can be divided from low frequency level (Level 1 = 15.625 Hz) to high frequency level (Level 6 = 300.0 Hz). The summation from level 0 to level 6 recovers completely the original turbulence velocities in Figs. 4 and 5 (Level 0 is not shown). In the waveform on the low frequency level (Levels 1 and 2) in the figures, the turbulence velocity of the spiral flow is much smaller than this of the typical turbulence flow. The waveform on the middle frequency levels (Levels 3 and 4) is slightly different, and then, high frequency level is the same.

Next, the auto-correlation on each level in Fig. 9 is obtained to classify which level is dominant in the spiral flow with

$$ R(r) = \frac{\sum \Omega(r) \Omega(r+r)}{\sqrt{\sum \Omega(r)^2} \cdot \sqrt{\sum \Omega(r+r)^2}} $$

(25)

r is the delay time from 0.0 to 512 ms. The auto-correlation is done binarization with threshold value +0.25 and -0.25 because the periodicity makes clear. In this study, the points over +0.25 and under -0.25 of the auto-correlation is mistuned to be high periodicity, and the points between -0.25 and +0.25 to be low periodicity. The binary auto-correlation is shown in Fig. 10. In this figure, the black part is under -0.25, and white part is over +0.25, which are high correlation parts. The gray part is between -0.25 and +0.25, which is low correlation part. From this figure, it is recognized that Level 3 is dominant in the spiral flow because the black part and the white part are shown repeatedly.

Fig. 7 Turbulence level on each frequency level

Table 2: Relation between Frequency and Wave number

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 5</th>
<th>Level 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave number</td>
<td>$1 \times 10^7$</td>
<td>$1 \times 10^9$</td>
<td>$1 \times 10^9$</td>
<td>$1 \times 10^9$</td>
<td>$1 \times 10^9$</td>
</tr>
<tr>
<td>Spiral flow [cm/s]</td>
<td>5.1</td>
<td>2.0</td>
<td>4.1</td>
<td>8.1</td>
<td>1.6</td>
</tr>
<tr>
<td>Typical flow [cm/s]</td>
<td>5.5</td>
<td>2.2</td>
<td>4.4</td>
<td>8.9</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Fig. 8 Multiresolution analysis (3D Display)

Fig. 9 Multiresolution analysis for turbulence velocity

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CONCLUSIONS
Time-frequency distributions of axial turbulence velocities of spiral pipe flow and typical turbulence flow have been clearly decomposed in a range from low frequency level (15.625 Hz) to high frequency level (500.0 Hz) by means of discrete wavelets transform. Also, the dominant level to be stabilized is classified. As a result, the following conclusions become clear.

(1) The time waveform on target level is able to extract by means of discrete wavelets transform and multiresolution because the orthonormal analyzing wavelets function composes and decomposes the original waveform. It is useful for analyzing the stability of spiral flow.

(2) The axial turbulence level in the under middle frequency levels (under Level 4 = 125.0 Hz) of spiral flow are extremely lower as compared with that of typical turbulence flow.

(3) Level 7 of spiral flow has high periodicity. It means that the axial stability of spiral flow is mainly dominated by Level 3.

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