A REPRESENTATION OF MAGNETIC HYSTERESIS BY FOURIER SERIES

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The magnetization characteristics are separated into the saturation and hysteresis properties by the Fourier series while the induction is sinusoidally varying with time. These properties yield the parameters of a Chua type model which is closely related with the Preisach type models.

1. Introduction

With the development of modern computers, numerical methods became available to calculate the magnetic fields of electromagnetic devices. In order to calculate accurately the fields, it is essential to work out a model which behaves just look like a true magnetic material. Even though a Chua type model is capable of any magnetization properties, a difficulty of its parameter determination has been pointed out.

In the present paper, a couple of parameters of a Chua type model is determined by means of a Fourier series. Also, it is shown that the saturation property is a function of the induction, and the hysteresis property is a function of the time derivative of induction. Furthermore, a Steinmetz type formula is given in the determination of the power loss in the materials is derived. Our formula suggests that the power loss is proportional to the square of peak value of induction within a certain range as long as the correct sinusoidal wave form of the induction is held.

2. Fourier analysis of magnetization characteristics

Fourier analysis is effectively used to clarify the harmonic content of the periodic waves. However, in the present paper, Fourier analysis is available to separate the periodic wave into the odd and even components.

When the induction $B$ is sinusoidally varying with time, the associated field intensity $H$ becomes to a non sinusoidal periodic wave. Fig. 1a shows a $B-H$ loop, and fig. 1b shows the wave forms of $B$ and $H$. By means of a Fourier series, the field intensity $H$ in fig. 1b is expanded into the sine and cosine series, that is

$$H = \sum_{n=1}^{\infty} H_n \sin(n \omega t) + \sum_{n=1}^{\infty} H_n \cos(n \omega t),$$

(1)

where $\omega$ denotes the angular velocity of $B$. Let $T$ be the period of $B$, then $\omega = (2\pi/T)$, and the Fourier coefficients $H_n, H_n$ for nth harmonics are given by

$$H_n = \frac{2}{T} \int_0^T H \sin(n \omega t) dt,$$

(2)

$$H_n = \frac{2}{T} \int_0^T H \cos(n \omega t) dt.$$
considering the relations of figs. 2a and b, it is possible to write the field intensity \( H \) as

\[
H = H_0 + H_s = f(B) + f_0(dB/dt),
\]

where \( f(B) \) and \( f_0(dB/dt) \) are denoting the single valued functions of \( B \) and \( dB/dt \), respectively. This means that the saturation property is a function of the induction \( B \) only, and the hysteresis property is a function of the time derivative of induction \( dB/dt \) only.

By defining the permeability \( \mu \) and hysteresis coefficient \( \chi \) as

\[
\mu = B/H, \quad \chi = (dB/dt)/H,
\]

the relation (6) is expressed by

\[
H = (1/\mu)B + (1/\chi)dB/dt. \tag{9}
\]

The expression (9) is known as a Saito's formula, and it has been reported that the hysteresis coefficient \( \chi \) is closely related with the Preisach's distribution function \( \Phi \).

Another important magnetization characteristic is the power loss in the materials. We derive here a Steinmetz type formula from a Chua type model (9) under the constraint of the correct sinusoidal wave form of induction.

Since the voltage per unit area is the time derivative of induction \( dB/dt \) and the current per unit length is the field intensity \( H \), the instantaneous power \( P_i \) per unit volume is given by

\[
P_i = N(dB/dt). \tag{10}
\]

We are enforcing the sinusoidally time varying induction \( B \) with angular velocity \( \omega \), so the mean power \( P_m \) of instantaneous power \( P_i \) is calculated by

\[
P_m = (\omega/2) H_0^2 R, \tag{11}
\]

where \( H_0 \) and \( R \) are the Fourier coefficient of 1st order even field in the relation (3) and the peak value of induction \( B \), respectively. Let \( \chi \) be the hysteresis coefficient for the fundamental wave defined as:

\[
\chi = \omega H_0/R, \tag{12}
\]

then the average power \( P_a \) can be expressed by

\[
P_a = (1/2\pi) \omega H_0^2 R. \tag{13}
\]

Expression (13) is a typical Steinmetz type formula [4] for the hysteresis loss. However, when the induction \( B \) and field intensity \( H \) in figs. 2a and b are including the oddy current effects, then expression (13) exhibits the losses due to the hysteresis as well as oddy current. The expression (13) suggests that the power \( P_{\text{loss}} \) must be proportional to the square of \( B_0 \) and \( \omega \) as long as \( \chi \) takes a constant value and the correct sinusoidal wave form of the induction \( B \) is held.

Although the correct experimental conditions have not been established, the experimental data by Prusty [5] have been fairly followed by formula (13) in our examination.

3. Conclusion

As shown above, an alternative formulation of a Chua type model has been carried out by means of a Fourier series. As a result, it has been clarified that the saturation property depends on the induction, on the other side, the hysteresis property depends on the time derivative of induction. Furthermore, it has been suggested that the power loss must be proportional to the square of peak value of induction within a certain range as long as the correct sinusoidal wave form of the induction is held.