ABSTRACT
Time-frequency distributions of axial turbulence velocities of spiral pipe flow and typical turbulence flow have been clearly decomposed in a range from low frequency level to high frequency level by means of discrete wavelets transform. As a result, the lower frequency levels (under Level 3) of the spiral flow are remarkably lower as compared with those of the typical turbulence flow. Moreover, the spiral flow is disintegrated by Level 3 to be stabilized from the autocorrelation. The originality of this paper lies in applying discrete wavelets transform and its autocorrelation analysis to analyzing the spiral flow stable motion in time-frequency dimension.

KEYWORDS: Discrete wavelet transform, Spiral flow, Frequency analysis, Turbulent velocity, Fluctuation level, Autocorrelation

INTRODUCTION
Spiral flow is a swirling flow with large free vortex region, high concentration in the axis and high stability (Hata et al. 1999). From the high stability characteristics, the spiral flow is useful for industrial applications such as optical cord installation in a small diameter pipeline with bends (Hori et al. 1999) and high performance pneumatic transportation without particle routing pipe inver (Takagi et al. 1997). The solids in the two-phase spiral pipe flow acquire stably their positive in a pipeline without large vortices. The motivation behind this work is to clarify the mechanism of the high stability in order to improve the spiral flow system. Time-frequency analysis is a suitable method to analyze the stability.

Recently, wavelet transform has been popular in time-frequency analysis instead of Fourier transform in mechanical engineering fields. The merits of the wavelet analysis is that it is be able to analyze the frequency not to create the false information. Wavelet Transform (Morlet 1989) is roughly classified with two types, which are continuous wavelets transform and discrete wavelets transform. The continuous wavelet transform has been generally used for time frequency analysis a vibration wave. For example, self-similarity of the inner structure of the jet (Eberly 1980), the breakdown of a large eddy and the successive branching of a large eddy structure in a plane jet (Li 1995), decompositions of Reynolds stresses in a jet (Goldstein 1995) and the multiple acoustic modes and the three layer instability (Walker 1995) were investigated. However, most of the researches on the time frequency analysis carried out the continuous wavelet transform. On the other hand, the discrete wavelet transform has been mainly used for picture image processing. The analysis enable to decompose and to compose of picture image data quantitatively because of the orthonormal transform. Salib applied this idea to analyzing the electromagnetic wave (Salib 1995). The originality of this paper lies in applying discrete wavelet transform and autocorrelation to each frequency level to analyzing the spiral flow stable motion. In this paper, as a first step to clarify the stability, time-frequency distribution of axial turbulence velocity of spiral pipe flow is decomposed from low frequency level to high frequency level by discrete wavelets transform and its autocorrelations. It is recognized which level is dominant to stabilize the spiral flow.

THEORY OF DISCRETE WAVELETS TRANSFORM

Basic concept of discrete wavelets transform is described using matrix expression aimed of integral expression. One dimensional input data matrix with four dimensions X is an analyzing wavelet matrix of Haar wave function W are used to simplify the expressions. For example, the input data matrix X is discrete velocity data with time. The wavelet transform matrix S that indicates wavelet spectrum is expressed by

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\[ x_i \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} y_i \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \]  
(1)

\[ x_i \begin{bmatrix} 1 & 2 & 1 \\ 2 & -2 & 0 \\ 0 & 0 & 0 \end{bmatrix} y_i \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \]  
(2)

\[ x_i \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  
(3)

\[ a \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  
(4)

or \( x_i = W X \)  
(5)

where, \( W_{1 \times 4}, W \) is a unit matrix and \( W \) is a transpose matrix of \( W \). The analyzing wavelet \( W_{1 \times 2} \) is an orthonormal wavelet.

In Eq. (1), the first element of the wavelet spectrum \( S \) shows a transform to obtain a mean value with a weight on the all input \( x_i \), \( c_{11} \) and \( c_{12} \). The second element in the wavelet spectrum \( D \) shows a transform to obtain a difference value between the first half and the latter half with a weight on the input data, \( (x_1 + x_2) / (x_1 - x_2) \). It means that this element includes the low frequency level of the input data. The third element \( c_2 \) shows a transform to obtain a difference value on the first half of the input data. \( c_3 \) and \( c_4 \) show a transform to obtain a difference value on the latter half. \( c_1 \) and \( c_2 \) show a transform to obtain a difference value on the entire data. \( c_3 \) and \( c_4 \) show a transform to obtain a difference value of the higher frequency level of the input data. Therefore, the input data is able to classify into a range from higher frequency level to lower frequency level. Because of orthonormal, the inverse discrete wavelet transform is expressed by:

\[ X = W^T Y \]  
(6)

Moreover, from Eq. (3), the input data \( X \) is decomposed by multiresolution. The matrix expression is:

\[ a \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]  
(7)

or \( X = W_Y^T Y \)  
(8)

where, \( S = \begin{bmatrix} S_0 \end{bmatrix} \), \( S_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \)  
(9)

\[ W = W_0, W_2 \]  
(10)

where, \( X \) is \( \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \)  
(11)

\[ Y_{1 \times 4} \]  
(12)

Moreover, from Eq. (11), the transform is continuously carried out by \( C \) and \( P \) with any operations to the difference values, \( W = \begin{bmatrix} \begin{bmatrix} S_0 \end{bmatrix} \end{bmatrix} \), \( S_0 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \)  
(13)

where, \( W_{1 \times 4} \) is a analyzing wavelet matrix test in Eq. (2). The wavelet spectrum \( S \) in Eq. (4) is \( W_{1 \times 4} \) in Eq. (14). In Eq. (13), \( \bar{S} \) indicates the mean value from \( S_0 \) to \( S_0 \). \( \bar{S} \) indicates the mean value from \( S_0 \) to \( S_0 \). \( \bar{S} \) indicates the mean value from \( S_0 \) to \( S_0 \). In Eq. (8), \( \bar{S} \) indicates the mean value from \( S_0 \) to \( S_0 \). In Eq. (13), \( \bar{S} \) indicates the difference value from \( S_0 \) to \( S_0 \). In Eq. (14). \( \bar{S} \) indicates the difference value from \( S_0 \) to \( S_0 \). In Eq. (13). From Eq. (14), the input data are transformed to the mean values and the difference values with high resolution levels by the discrete wavelet transform. The input data are divided into a range from high frequency to low frequency. From Eq. (14), the inverse wavelet transform is.

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\[ X = |W|X_0 \]
\[ |W|P_x_1C_x_1 + |W|P_x_2C_x_2 + |W|P_x_3C_x_3 + |W|P_x_4C_x_4 \]
\[ = |W|P_x_5C_x_5 + |W|P_x_6C_x_6 + |W|P_x_7C_x_7 \]
\[ = |W|P_x_8C_x_8 + |W|P_x_9C_x_9 \]
\[ = (19) \]

From Eq. (18), the multiformation is:
\[ X = |W|X_0 \]
\[ = |W|X_1 + |W|X_2 + |W|X_3 + |W|X_4 + |W|X_5 + |W|X_6 \]
\[ = |W|X_7 + |W|X_8 + |W|X_9 \]
\[ = |W|X_{10} \]
\[ = (20) \]

Where,
\[ S_0 = [S_0, S_1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \]
\[ S_1 = [0, 0, 0, 0, D, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \]
\[ S_2 = [0, 0, 0, 0, 0, 0, D, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0] \]
\[ S_3 = [0, 0, 0, 0, 0, 0, 0, 0, D, 0, 0, 0, 0, 0, 0, 0, 0, 0] \]
\[ S_4 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, D, 0, 0, 0, 0, 0, 0, 0] \]
\[ S_5 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, D, 0, 0, 0, 0, 0] \]
\[ S_6 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, D, 0, 0, 0] \]
\[ S_7 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, D, 0] \]
\[ S_8 = [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, D] \]
(21)

In the case of systems with fixed data and fourth Doutěechies, multiformation indicates from Level 0 to Level 3. In general, in the case that input data is In and Doutěechies function is End (N+4), the algorithm to obtain levels is shown in Fig. 1. The final wavelet spectrum is obtained after the wavelet transform in Eq. (14) continues until the number of final summation elements is In three.

![Algorithm of discrete wavelet transform](image)

**Fig. 1 Algorithm of discrete wavelet transform**

**EXPERIMENTS**

**Nozzles to Produce Spiral Flow**

The nozzle to produce the spiral flow is designed with an annular slit connecting to a conical cylinder as shown in Fig. 2 [Hosni 1998]. Pressurized air is forced through the sides of the device into the buffer area, and then through the annular slit into a vertical pipe entrance. The suction form is generated at the back of the nozzle by Cawna's effect. The annular flow, passing through the conical cylinder, develops a spiral structure with a deeper axial velocity and an azimuthal velocity distribution, even if it is not applied tangentially. Vapored water at a tracer of LDV are sucked into the nozzle from the back of the nozzle. An ejector is used to generate the typical turbulence flow.

![Experimental equipment](image)

**Fig. 3 Experimental equipment**

**Experimental Equipment, Method & Conditions**

The experimental equipment consisted of a vertical acrylic pipe, the nozzle to produce the spiral flow and an air compressor as shown in Fig. 2. The inside diameter of the vertical pipe was 41.0 mm. A LDV probe is set up at the side of the vertical pipe, at 1.0 m from the air supply port to measure the axial velocity at the center of the pipe. The He-Ne Laser power of LDV was 10 mW, and the probe pick up the reflected wave from the tracer. The air flow rate was 1.98 X 10^6 m^3/s. The mean velocity of the air flow in the vertical pipe calculated from the flow rate was 1.50 m/s. Reynolds number calculated from the mean velocity was about 6,200.

The reflected wave passed through a timer unit connecting to LDV probe for 1 (0.001 Hz) pick-up interval. The signals of the reflected wave were captured for about 3 seconds in a computer system connecting the timer unit. The data sampling velocity data were 40(24+73). The computer system has high pass and low pass filters that reduce signals under 0.625 m/s and over 0.625 m/s as noise. The pick up point is one point where the center of the pipe as a first step study. The time mean velocities and turbulence levels of the spiral flow and typical turbulence flow are compared.

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Experimental Results

The velocities of the spiral flow and typical turbulence flow at the center of the pipe are obtained with LDV. The turbulence level is defined as:

\[ v'_r = \sqrt{\frac{1}{T} \int_0^T (v_r - \bar{v}_r)^2} \]  

(22)

Where, \( n \) is the sampling frequency number, \( \bar{v}_r \) is the time mean velocity and \( v_r \) is a velocity in a time. The time mean velocity and the turbulence level are shown in Table 1. From this table, the time mean velocity of the spiral flow is higher that of of typical turbulence flow by about 9% even though the AH flow rate is the same (Harris 1990). That is because the axial velocity of the spiral flow is steeper than that of the typical turbulence flow. Also, the turbulence level of the spiral flow is much lower than the typical turbulence flow by about 10%. It means the spiral flow is a stable flow in an axial direction. The normalized axial turbulence velocities \( v'_r / \bar{v}_r \) are shown in Figs. 4 and 5. These figures are analyzed in the next section.

Table 1: Time-mean velocity and turbulence level

<table>
<thead>
<tr>
<th>Time-mean velocity</th>
<th>Turbulence level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral Flow</td>
<td>1.71 m/s</td>
</tr>
<tr>
<td>Typical Turbulence</td>
<td>1.77 m/s</td>
</tr>
</tbody>
</table>

Fig. 4 Axial velocity of spiral flow

Fig. 5 Axial turbulence of typical turbulence flow

ANALYSIS AND DISCUSSION

Analysis Method

The normalized axial turbulence velocities in Figs. 4 and 5 are analyzed by discrete wavelet transform and its autocorrelation. This wavelet analysis consists of three steps. Firstly, the d24 (\( 2^{12} \)) sampling data of the axial turbulence velocities are put into the matrix X in Eq. (9). The matrix X is transformed to the wavelet spectrum S in Eq. (14). Next, the multiresolution analysis is carried out, that is, each part of the spectrum is inversely transformed to multiresolution levels by means of the discrete inverse wavelet transform in Eq. (20). Finally, to recognize which level is dominant for the spiral flow stability, autocorrelation of each level is obtained.

Twentieth Daubechies function is used as an analyzing wavelet function. Twentieth Daubechies function has twenty coefficients from \( c_1 \) to \( c_{20} \) in the first ten in Eq. (6). Twenty coefficients from \( c_1 \) to \( c_{20} \) in the second line in Eq. (15). In the case of twentieth Daubechies function and d24 (\( 2^{12} \)) input data, the multiresolution classes to seven levels as shown in Eq. (21).

\[ X = [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] + [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] + [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] + [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] + [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] + [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] + [w_1 w_2 w_3 w_4 w_5 w_6 w_7 w_8 w_9 w_{10}] 

(21)

\( w_0 \) indicates the five times operation to obtain Daubechies matrix from a matrix C in Eq. (6). The coefficients of twentieth Daubechies function are shown in Fig. 6. a axis shows the coefficients from \( w_1 \) to \( w_{20} \) in the second line of C matrix in Eq. (6). Therefore, 1 in a axis indicates \( c_1 \) to 20 indicates \( c_{19} \) and 20 indicates \( c_{20} \).

Fig. 6 Coefficients of twentieth Daubechies function

Turbulence Level on Each Frequency Level

To clarify the difference between wavelets transform and Fourier transform, the axial turbulence level on each frequency level defined in Eq. (24) is calculated before indicating the wavelets analysis.

\[ v'_r = \sqrt{\frac{1}{T} \int_0^T (v_r - \bar{v}_r)^2} \]  

(24)

Where, \( n \) is a frequency level, \( \bar{v}_r \) is a time-mean velocity and \( v_r \) is a turbulence velocity in a time on each wavelet level. The \( v'_r \) indicates a kind of normalized turbulence level obtained by Fourier transform. The mean turbulence velocity on each frequency level \( \bar{v}_r \) is not zero. The turbulence level on each frequency level is shown in Fig. 7 (Level 0 is not shown). From this figure, the turbulence levels of the spiral flow on all levels are lower than that of typical turbulence flow. Mainly, the level from Level 1 to Level 4 remarkably different. Both turbulence levels have peaks at Level 3.

The relation between the frequency level and the wave number is shown in Table 2. If Kolmogorov wave number \( k_r \) is assumed to be \( 10^7 \) order, the wave number range normalized with \( k_r \) is flow \( 10^{-10} \) to \( 10^{-7} \). It means that the range includes the energy contain range and the inertial range. The position of the peaks are reasonable from Krause-Gorlov theory.

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Fig. 7 Turbulence level on each frequency level

Table 2 Relative between Frequency and Wave number

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
<th>Level 5</th>
<th>Level 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave number</td>
<td>2.2</td>
<td>1.6</td>
<td>2.0</td>
<td>3.3</td>
<td>3.7</td>
</tr>
<tr>
<td>Wave number</td>
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<td>2.0</td>
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<td>2.0</td>
<td>3.3</td>
<td>3.7</td>
</tr>
</tbody>
</table>

Wavelets Analysis Results & Discussion

Transforming inversely each level of the wavelets spectrum indicates multiresolution as shown in Eq.(20). Fig. 8(A) shows the multiresolution of the spiral flow, and Fig. 8(B) shows the multiresolution of the typical turbulence flow in three dimension display. From Fig. 8, it is recognized that time and frequency level is simultaneously analyzed. To clarify the each frequency level, Fig. 8 is displayed in two dimensions as shown in Fig. 9. From this multiresolution, the spectrum can be divided from low frequency level (Level 1) to high frequency level (Level 6). The summation from level 0 to level 6 recovers the original turbulence velocities in Figs. 4 and 5 (Level 0 is not shown). In the waveforms on the low frequency level (Level 1 and 2) in the figures, the turbulence velocity of the spiral flow is much smaller than that of the typical turbulence flow. The waveforms on the middle frequency levels (Levels 3 and 4) are slightly different, and then, high frequency level is the same.

Next, the autocorrelation on each level in Fig. 9 is obtained to classify which level is dominant in spiral flow with

\[
\gamma(t) = \frac{\sum_{i=1}^{N} x(i) \cdot x(i+t)}{\sqrt{\sum_{i=1}^{N} x(i)^2} \cdot \sqrt{\sum_{i=1}^{N} x(i+t)^2}}
\]

in the delay time from 0 to 512 ms. The autocorrelation is done binarization with threshold value +0.25 and -0.25 because the periodicity makes clear. In this study, the points over +0.25 and under -0.25 of the autocorrelation is assumed to be high periodicity, and the points between -0.25 and +0.25 to be low periodicity. The binary autocorrelation is shown in Fig. 10. In this figure, the black part is under -0.25, and white part is over +0.25, which are high correlation parts. The gray part is between -0.25 and +0.25, which is low correlation part. From this figure, it is recognized that Level 3 is dominant in the spiral flow because the black part and the white part are shown repeated.

Fig. 8 Multiresolution analysis (3D Display)

(A) Spiral flow

(B) Typical turbulence flow

Fig. 8 Multiresolution analysis (3D Display)

(A) Spiral flow

(B) Typical turbulence flow

Fig. 9 Multiresolution analysis for turbulence velocity

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CONCLUSIONS

Time-frequency distributions of axial turbulence velocities of spiral pipe flow and typical turbulent flow have been clearly decomposed in a range from low frequency level to high frequency level by means of discrete wavelets transform. Also, the dominant level to be analyzed is classified. As a result, the following conclusions become clear.

1. The time waveform on target level is quite to extract by means of discrete wavelets transform and multiresolution because the fundamental analyzing wavelets function composed and decomposes the original waveform. It is useful for analyzing the stability of spiral flow.

2. The axial turbulence level in the middle frequency levels (under Level 4) of spiral flow are extremely lower to compared with that of typical turbulent flow.

3. Level 3 of spiral flow has high periodicity. It means that the axial stability of spiral flow is mainly dominated by Level 3.

ACKNOWLEDGEMENTS

The authors are wished to acknowledge the considerable assistance of Mr. T. Kamiyama and Mr. K. Kato in Nihon University.

REFERENCES


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