PROGRAM

PNEUMATIC AND HYDRAULIC CONVEYING SYSTEMS II

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ABSTRACT
Time-frequency distributions of axial turbulence velocities of spiral pipe flow and typical turbulence flow have been clearly decomposed in a range from low frequency level to high frequency level by means of discrete wavelets transform. As a result, the lower frequency levels (under Level 4) of the spiral flow are extremely lower as compared with those of the typical turbulence flow. Moreover, the spiral flow is dominated by Level 3 to be stabilized from the autocorrelation. The originality of this paper lies in applying discrete wavelets transform and its autocorrelation analysis to analyzing the spiral flow stable motion in time-frequency dimension.

KEYWORDS: Discrete wavelets transform, Spiral flow, Frequency analysis, Turbulence velocity, Fluctuation level, Autocorrelation

1. INTRODUCTION
Spiral flow is a swirling flow with large free vortex region, high concentration to the axis and high stability [1]. From the high stability characteristics, the spiral flow is useful for industrial applications such as optical and installation in a small diameter pipeline with bends[2] and high performance pneumatic transportation without particles touching pipe inner wall[3]. The solids in the two-phase spiral pipe flow acquire their position in a pipeline without large vibration. The motivation behind this work is to clarify the mechanism of the high stability in order to improve the spiral flow system. Time-frequency analysis is a suitable method to analyze the stability as a first step.

Recently, wavelets transform has been popular for time-frequency analysis instead of Fourier transform in mechanical engineering fields. The merits of the wavelets analysis is to be able to analyze the frequency not to erase the time information. Wavelets transform [4] is roughly classified with two types, which are continuous wavelets transform and discrete wavelets transform. The continuous wavelets transform has been generally used for time frequency analysis in vibration wave. For example, self-similarity of the inner structure of the jet [5], the breakdown of a large eddy and the successive branching of a large eddy structure in a plane jet [6], decomposition of Reynolds stress in a jet [7], and the multiple acoustic modes and the shear layer instability [8] were investigated.

However, most of the researchers on the time-frequency analysis carried out the continuous wavelets transform. On the other hand, the discrete wavelets transform has been mainly used for picture image processing. The analysis enables to decompose and to compress picture image data quantitatively because of the orthonormal transform. Saito applied this idea to analyzing the electromagnetic wave[9].

The originality of this paper lies in applying discrete wavelets transform and autocorrelation to each frequency level to analyzing the spiral flow stable motion. In this paper, as a first step to clarify the stability, time-frequency distribution of axial turbulence velocity of spiral pipe flow is decomposed from low frequency level to high frequency level by discrete wavelets transform and its autocorrelation. It is recognized which level is dominant to stabilize the spiral flow.

2. THEORY OF DISCRETE WAVELETS TRANSFORM
2.1 Basic Concept Using Simple Base Function
Basic concept of discrete wavelets transform is described using matrix expression instead of integral expression. One dimensional input data matrix with four elements X and an analyzing wavelets matrix of Haar base function W are used to simplify the expression. For example, the input data matrix X is discrete velocity data with time. The wavelets transform matrix S that indicates wavelets spectrum is expressed by

1
\[ S_i = \begin{bmatrix} 1 & 1 & 1 & 1 \\ d_k & -\frac{1}{2d_k} & 1 & -1 \\ d_l & 1 & 1 & -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \] \hspace{1cm} \text{(1)}

or \[ S = W \cdot X \] \hspace{1cm} \text{(2)}

Where, \( W^T \cdot W \) is the transpose of \( W \). The analyzing wavelets matrix is an orthonormal.

In Eq. (1), the first element in the wavelets spectrum \( S_1 \) shows a transform to obtain a mean value with a weight on the all input data. \( a+b+c+d \). The second element in the wavelets spectrum \( D_1 \) shows a transform to obtain a difference value between the fast half and the latter half with a weight on the input data, \( (a-b)-(c-d) \). It means that this element includes the lower frequency level of the input data. The third element \( D_2 \) shows a transform to obtain a difference value on the first half of the input data, \( (a-b) \). The fourth element \( D_3 \) shows a transform to obtain a difference value on the latter half, \( (c-d) \). The third and forth elements include the higher frequency level of the input data. Therefore, the input data is able to classified to a range from higher frequency level to lower frequency level. Because of orthonormal, the inverse discrete wavelets transform is expressed by:

\[ X = W^T \cdot S \] \hspace{1cm} \text{(3)}

Moreover, from Eq. (3), the input data \( X \) is decomposed by multiple resolution. The matrix expression is:

\[ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ d_k & -\frac{1}{2d_k} & 1 & -1 \\ d_l & 1 & 1 & -1 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_1 \\ 0 \\ 0 \\ s_4 \end{bmatrix} \] \hspace{1cm} \text{(4)}

or \[ X = W_1^T \cdot S_1 + W_2^T \cdot S_2 + W_3^T \cdot S_3 \] \hspace{1cm} \text{(5)}

Where, \( S_0 = [0, 0, 0, 0]^T \)
\( S_1 = [0, 0, 0, 0]^T \)
\( S_2 = [0, 0, 0, 0]^T \)
\( W_1^T \cdot S_1, W_2^T \cdot S_2, \) and \( W_3^T \cdot S_3 \) are called Level 0, Level 1 and Level 2, respectively.

### 2.2 Generalization of Discrete Wavelets Transform

Many orthonormal wavelets analyzing function are found [4]. The basic concept of the discrete wavelets transform is generated by using four Daubechies function (N=4). The analyzing wavelets matrix is also an orthonormal function.

The analyzing wavelets matrix \( W \) is acquired by a cascade algorithm on the basis of a function matrix \( C \). The matrix \( C \) is shown in Eq. (6):

\[ C = \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \\ c_1 & c_0 & -c_2 & -c_3 \\ c_2 & c_3 & c_0 & -c_1 \\ c_3 & -c_2 & c_1 & c_0 \end{bmatrix} \] \hspace{1cm} \text{(6)}

\[ \begin{bmatrix} c_0 & c_1 & c_2 & c_3 \end{bmatrix} \] \hspace{1cm} \text{(7)}

Where, \( C^T \cdot C = I \). The first line in Eq. (6) is called scaling coefficients and second line is called wavelets coefficients. Forth Daubechies function (N=4) has four coefficients in a line. The first line shows a transform to obtain a mean value with weights of \( c_0, c_1, c_2 \) and \( c_3 \) on the input data. The second line shows a transform to obtain a difference value with weights of \( c_0, c_1, c_2 \) and \( c_3 \) on the input data. The third line shows a transform to translate the first line by two steps. The fourth line is a transform to do the second line by two steps. Eqs. (7) and (8) show the transformed values are zero when the input are constant or are simply increased. To explain easily the process to acquire the analyzing wavelets matrix \( W \) from \( C \), the matrix \( X \) is assumed as one dimensional 16 elements.

\[ X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}] \] \hspace{1cm} \text{(9)}

From Eqs. (6) and (9), the transformed matrix \( X^T \) in

\[ X^T = C^T \cdot X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}] \] \hspace{1cm} \text{(10)}

Where, \( C \) is 16X16 matrix of \( C \). The element \( s \) indicates the mean value and the element \( d \) indicates the difference value. The elements in the matrix \( X^T \) are replaced by a matrix \( P_{16} \).

\[ P_{16} \cdot X^T = P_{16} \cdot C_{16} \cdot X = [x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}] \] \hspace{1cm} \text{(11)}

Where, \( P_{16} \) is defined as

\[
\begin{align*}
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]

\[
\begin{align*}
\end{align*}
\]
Moreover, from Eq. (11), the transform is continuously carried out by C and P without any operations to the difference values,

$$W^0X = [S_x; S_y; S_x; D_x; D_y; D_x; D_y; d_x; d_y; d_x; d_y]$$  \hspace{1cm} (13)

$$S = W^0X$$  \hspace{1cm} (14)

Where,

$$W^0 = (P_0^0 C_0^0)$$  \hspace{1cm} (15)

$$W^{(k)} = (P_0^{(k)} C_0^{(k)})$$  \hspace{1cm} (16)

$$n = [n_0; n_0; 0; 0; 0; 0; 0; 0; 0; 0]$$  \hspace{1cm} (17)

$$W^0$$ is a analyzing wavelets matrix that is W in Eq. (2). The wavelets spectrum S in Eq. (2) is W^0X in Eq. (14). In Eq. (13), S indicates the mean value from s1 to s2 in Eq. (11). S_x indicates the mean value from s1 to s2 that translate by two steps. D_x indicates the difference value from s1 to s2. In Eq (14), S indicates the mean value from S_x to S_y in Eq. (13). D_y indicates the difference value from S_y to S_x in Eq. (13). From Eq. (14), the input data are transformed to the mean values and the difference values with valuable resolution levels by the discrete wavelets transform. The input data are divided into a range from high frequency to low frequency.

From Eq. (14), the inverse wavelets transform is,

$$X = [W^0]^T S$$  \hspace{1cm} (18)

$$[W_0^{(k)}]^T S_0 = (P_0^{(k)} C_0^{(k)}) (P_0^{(k)} C_0^{(k)}) (P_0^{(k)} C_0^{(k)}) (P_0^{(k)} C_0^{(k)})$$  \hspace{1cm} (19)

From Eq. (18), the multiresolution is,

$$X = [W^0]^T S = [W^0]^T S_x + [W^0]^T S_y + [W^0]^T S_0$$  \hspace{1cm} (20)

Where,

$$S_x = [S_x; S_y; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0]$$  \hspace{1cm} (21)

$$S_y = [0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0 ; 0]$$  \hspace{1cm} (22)

In the case of sixteen input data and fourth wavelets, multiresolution indicates from Level 0 to Level 3. In general, in the case that input data is 2^n and wavelets function is 6b (N-1), the algorithm to obtain level is shown in Fig. 1. The final wavelets spectrum is obtained after the wavelet transform in Eq (14) continues until the number of final summation elements is less than k.

![Fig. 1 Algorithm of discrete wavelets transform](image_url)
3. EXPERIMENTS

3.1 Nozzle to Produce Spiral Flow

A nozzle to produce the spiral flow is designed with an annular slit connecting to a conical cylinder as shown in Fig. 2 [10]. Pressurized air is forced through the sides of the device into the buffer area, and then through the annular slit into a vertical pipe entrance. The suction force is generated at the back of the nozzle by Coanda effect. The annular flow, passing through the conical cylinder, develops a spiral structure with a steeper axial velocity and an azimuthal velocity distribution, even if it is not applied tangentially. Vaporized water as a tracer of LDV are sucked into the nozzle from the back of the nozzle. An ejector is used to generate the typical turbulence flow.

3.2 Experimental Equipment, Method & Conditions

The experimental equipment consisted of a vertical acrylic pipe, the nozzle to produce the spiral flow and an air compressor as shown in Fig. 3. The inside diameter of the vertical pipe was 41.0 mm. A LDV probe is set up at the side of the vertical pipe at 1.0 m from the air supply part to measure the axial velocity at the center of the pipe. He-Ne Laser power of LDV was 10 mW, and the probe picked up the reflected wave from the tracer. The air flow rate was 1.98 X 10^{-5} m³/s. The mean velocity of the air flow in the vertical pipe calculated from the flowrate was 1.50 m/s. Reynolds number is calculated from the mean velocity was about 4,200.

The reflected wave pass though a timer unit connecting to LDV probe with 1ms (1,000 Hz) pick-up interval. The signals of the reflected wave were counted for about 5 seconds in a counter system connecting to the timer unit. The discrete sampling velocity data were n=1024=2^{10}. The counter system has high pass and low pass filters that reduce signals under 0.625 m/s and over 6.25 m/s as noise. The pick up point is one point where is the center of the pipe as a first step study. The time mean velocities and turbulence levels of the spiral flow and typical turbulence flow are compared.

![Fig. 3 Experimental equipment](image)

3.3 Experimental Results

The velocities of the spiral flow and typical turbulence flow at the center of the pipe are obtained with LDV. The turbulence level is defined as,

\[ v' = \frac{1}{n} \sum \left( \sqrt{(v - \bar{v})^2} / \bar{v} \right) \]  (22)

Where, \( n \) is the sampling velocity number, \( \bar{v} \) is the time mean velocity, \( v \) is a pick-up time and \( v' \) is 4th pick-up velocity in a pick up time. The time mean velocity and the turbulence level are shown in Table 1. From this table, the time mean velocity of the spiral flow is higher than that of typical turbulence flow by about 9% even though the air flow rate is the same [1]. That is because the axial velocity of the spiral flow is steeper than that of the typical turbulence flow. Also, the turbulence level of the spiral flow is much lower than the typical turbulence flow by about 10%. It means the spiral flow is a stable flow in an axial direction.

The normalized axial turbulence velocities in 4th pick-up time with each mean velocity \( v' = (v - \bar{v}) / \bar{v} \) are shown in Figs. 4 and 5. These figures are analyzed in the next section.
### Table 1: Time-mean velocity and turbulence Level

<table>
<thead>
<tr>
<th>Flow Type</th>
<th>Time mean velocity $\langle v \rangle$</th>
<th>Turbulence level $\sigma_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spiral Flow</td>
<td>1.93 m/s</td>
<td>0.06616</td>
</tr>
<tr>
<td>Typical Turbulce Flow</td>
<td>1.77 m/s</td>
<td>0.07336</td>
</tr>
</tbody>
</table>

![Amplitude vs. Time](image1.png)  
![Amplitude vs. Time](image2.png)

**Fig. 4** Axial turbulence velocity of spiral flow  
**Fig. 5** Axial turbulence velocity of typical turbulence flow

### 4. ANALYSIS AND DISCUSSION

#### 4.1 Analysis Method

The normalized axial turbulence velocities in Figs. 4 and 5 are analyzed by discrete wavelet transforms and its autocorrelation. This wavelet analysis consists of three steps. Firstly, the 1024 ($2^{10}$) sampling data of the axial turbulence velocities are put into the matrix $X$ in Eq. (9). The matrix $X$ is transformed to the wavelets spectrum $S$ in the algorithm from Eq. (10) to Eq. (14). Next, the multiresolution analysis is carried out, that is, each part of the spectrum is inversely transformed to multiresolution levels by means of the discrete inverse wavelets transform in Eq. (20). Finally, autocorrelation of each level is obtained to recognize which level is dominant for the spiral flow stability.

Twentieth Daubechies function is used as an analyzing wavelet function. Twentieth Daubechies function has twenty coefficients from $c_0$ to $c_{20}$ in the first line in Eq. (6), twenty coefficients from $c_{21}$ to $c_{40}$ in the second line in Eq. (6). In the case of twentieth Daubechies function and 1024 ($2^{10}$) input data, the multiresolution classifies to seven levels as shown in Eq. (23).

$$
X = [W^{(0)}]S = [W^{(0)}]S_s + [W^{(0)}]S_t + [W^{(0)}]S_1 + [W^{(0)}]S_2 + [W^{(0)}]S_3 + [W^{(0)}]S_4 + [W^{(0)}]S_5 + [W^{(0)}]S_6 + [W^{(0)}]S_7
$$

$W^{(0)}$ indicates the five times operation to obtain Daubechies matrix from a matrix $C$ in Eq. (6). The coefficients of twentieth Daubechies function are shown in Fig. 6. $x$ axis shows the coefficients from $c_{20}$ to $c_0$ in the second line of $C$ matrix in Eq. (6). Therefore, 1 in $x$ axis indicates $c_{20}$, 2 in $x$ axis indicates $c_{19}$, and 20 indicates $c_0$.

![Coefficient values](image3.png)

**Fig. 6** Coefficients of twentieth Daubechies function

#### 4.2 Turbulence Level on Each Frequency Level

To clarify the difference between wavelets transform and Fourier transform, the axial turbulence level on each frequency level defined in Eq. (24) is calculated before indicating the wavelets analysis.

$$
\nu' = \frac{1}{\pi L} \sum_{l=0}^{L-1} \left[ |P(l)| - |P_{\text{Fourier}}(l)| \right]
$$

Where, the capital $P_l$ indicate velocities on each wavelet level, in detail, $P_{\text{Fourier}}$ is a time mean velocity and $P_l$ is a normalized turbulence velocity in 4th pick-up time on each wavelet level. $\nu'$ indicates a normalized turbulence level
in a wavelet level which includes a power spectrum obtained by Fourier transform. \( V'_t \) is not zero in the strict sense. The normalized turbulence level on each wavelet level \( \psi \) is shown in Fig. 7 (Level 0 is not shown). From the other view, Fig. 7 is a kind of power spectrum by Fourier Transform. From this figure, the turbulence levels of the spiral flow on all levels are lower than those of typical turbulence flow. Mainly, the level from Level 1 to Level 4 are remarkably different. Both turbulence levels have peaks at Level 3. The peak results from the energy-certain range and the inertia range from Kolmogorov theory.

**Fig. 7 Turbulence level on each frequency level**

4.3 Wavelets Analysis Results & Discussion

Transforming inversely each level of the wavelets spectrum indicates multiresolution as shown in Eq.(20). Fig. 8(A) shows the multiresolution of the spiral flow, and Fig. 8(B) shows the multiresolution of the typical turbulence flow in three dimension display. They show the relation among the time, wavelet level and normalized turbulence velocity \( V'_t \). From Fig. 8, it is recognized that time and frequency level is simultaneously analyzed. To clarify the each frequency level, Fig. 8 is displayed in two dimension as shown in Fig. 9. From this multiresolution, the spectrum can be divided from low frequency level (Level 1) to high frequency level (Level 6). The summation from level 0 to level 6 recovers completely the original turbulence velocities in Figs. 4 and 5 (Level 0 is not shown). In the waveform on the low frequency level (Levels 1 and 2) in the figures, the turbulence velocity of the spiral flow is much smaller than this of the typical turbulence flow. The waveform on the middle frequency levels (Levels 3 and 4) is slightly different, and then, high frequency level is the same. Next, the autocorrelation on each level in Fig. 9 is obtained to classify which level is dominant in the spiral flow with

\[
R(r) = \frac{\langle V'_t(r) \cdot V'_t(0) \rangle}{\langle V'_t(0)^2 \rangle}
\]

\( r \) is the delay time from 0.0 to 0.512 ms. The autocorrelation is done binarization with threshold value +0.25 and -0.25 because the periodicity makes clear. In this study, the points over +0.25 and under -0.25 of the autocorrelation is assumed to be high periodicity, and the points between -0.25 and +0.25 to be low periodicity. The binary autocorrelation is shown in Fig. 10. In this figure, the black part is under +0.25, and white part is over +0.25, which are high correlation parts. The gray part is between -0.25 and +0.25, which is low correlation part. From this figure, it is recognized that Level 3 is dominant in the spiral flow because the **black part** and the **white part** are shown repeatedly.

**Fig. 8 Multiresolution analysis (3D Display)**

(A) Spiral flow

(B) Typical turbulence flow
Fig. 9  Multiresolution analysis for turbulence velocity

(A) Spiral flow  (B) Typical turbulence flow

Fig. 10  Binary autocorrelation
5. CONCLUSIONS

Time-frequency distributions of axial turbulence velocities of spiral pipe flow and typical turbulence flow have been clearly decomposed in a range from low frequency level to high frequency level by means of discrete wavelets transform. Also, the dominant level to be stabilized is classified. As a result, the following conclusions become clear.

1. The time waveform on target level is able to extract by means of discrete wavelets transform and multiresolution because the orthonormal analyzing wavelets function decomposes and decomposes the original waveform. It is useful for analyzing the stability of spiral flow.

2. The axial turbulence level in the under middle frequency levels (under Level 4) of spiral flow are extremely lower as compared with that of typical turbulence flow.

3. Level 3 of spiral flow has high periodicity. It means that the axial stability of spiral flow is mainly dominated by Level 3.

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