JSAEM Studies in Applied Electromagnetics and Mechanics, 11

APPLICATIONS OF ELECTROMAGNETIC PHENOMENA IN ELECTRICAL AND MECHANICAL SYSTEMS

Proceedings of
The First Japanese-Australian Joint Seminar on Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
16-17 March 2000, Adelaide, Australia

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Applications of Electromagnetic Phenomena in Electrical and Mechanical Systems
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Current visualization by a generalized vector sampled pattern matching method

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ABSTRACT: The generalized vector sampled pattern matching (GVSPM) method has been proposed to solve the ill-posed inverse problems. Major difficulty of the mentioned problems has been removed by GVSPM. This paper presents one of the methodologies to check up the validity of the GVSPM solutions. A key idea was to extract the common parts among the independently evaluated solutions. Each of the GVSPM solutions was evaluated from the orthogonal x-, y- and z- components of the measured field vectors. Three kinds of the current distributions were evaluated by means of GVSPM.

1 INTRODUCTION

Visualization of the current distributions on printed circuit boards is of paramount importance for searching the fault points and checking up the electromagnetic compatibility of modern electronic devices [1,2]. In order to visualize the current distributions without destruction or decomposition of the electronic devices, a kind of inverse problem can be used. Available information from the outside of an electronic device is only a set of locally measured electromagnetic fields. It means, the electromagnetic field sources have to be evaluated from the locally measured electromagnetic fields. Most of the inverse problems are reduced into solving an ill-posed linear system of equations.

2 SOLUTION STRATEGY OF AN ILL-POSED SYSTEM

2.1 Ill-posed linear system of equations

Let us consider a linear system of equations:

$$\mathbf{Y} = C\mathbf{X}, \tag{1}$$

where **Y** and **X** are the n^{th} order input and m^{th} order output vectors, respectively. The matrix C is an n by m rectangular system matrix. Eq.(1) can be rewritten as:

$$\mathbf{Y} = \sum_{i=1}^{m} x_i \mathbf{C}_i, \tag{2}$$

$$\mathbf{X} = [x_1 \quad x_2 \quad . \quad x_m]^T, C = [\mathbf{C}_1 \quad \mathbf{C}_2 \quad . \quad \mathbf{C}_m]$$

Further modification to Eq. (2):

$$\frac{\mathbf{Y}}{|\mathbf{Y}|} = \sum_{i=1}^{m} x_i \frac{|\mathbf{C}_i|}{|\mathbf{Y}|}\frac{\mathbf{C}_i}{|\mathbf{C}_i|} \; or \; \mathbf{Y}' = C'\mathbf{X}'. \tag{3}$$

Eq.(3) means that the normalized input vector **Y'** can be given by a linear combination of the weighted solutions $x_1|\mathbf{C}_1|/|\mathbf{Y}|, x_2|\mathbf{C}_2|/|\mathbf{Y}|,.., x_m|\mathbf{C}_m|/|\mathbf{Y}|$ with the normalized column vectors $\mathbf{C}_1/|\mathbf{C}_1|, \mathbf{C}_2/|\mathbf{C}_2|,.., \mathbf{C}_m/|\mathbf{C}_m|$.

2.2 Objective function

Eq.(2) means that the input vector **Y** can be given by means of a linear combination of the column vector $\mathbf{C}_i$. Therefore, according to the Cauchy-Schwarz relationship [4], an angle between the input vectors of **Y** and of $C\mathbf{X}^{(k)}$, given in terms of the k^{th} iterative solution $\mathbf{X}^{(k)}$, is defined by:

$$f(\mathbf{X}^{(k)}) = \frac{\mathbf{Y}}{|\mathbf{Y}|}\cdot\frac{C\mathbf{X}^{(k)}}{|C\mathbf{X}^{(k)}|} = \mathbf{Y}'\cdot\frac{C'\mathbf{X}'^{(k)}}{|C'\mathbf{X}'^{(k)}|}, \tag{4}$$

So that

$$f\left(\mathbf{X}^{(k)}\right) \to 1, \tag{5}$$

The solution vector $\mathbf{X}^{(k)}$ satisfies the Eq.(3), i.e.,

$$\mathbf{Y}' = C'\mathbf{X}'^{(k)}. \tag{6}$$

When an initial solution vector $\mathbf{X}'^{(0)}$ is given by:

$$\mathbf{X}'^{(0)} = C'^{T}\,\mathbf{Y}', \tag{7}$$

the first deviation to the normalized input vector $\mathbf{Y}'$ becomes:

$$\Delta\mathbf{Y}'^{(1)} = \mathbf{Y}' - \frac{C'\mathbf{X}'^{(0)}}{\left|C'\mathbf{X}'^{(0)}\right|}. \tag{8}$$

By means of Eqs.(7) and (8), the k[th] iterative solution vector $\mathbf{X}'^{(k)}$ is given by:

$$\begin{aligned}\mathbf{X}'^{(k)} &= \mathbf{X}'^{(k-1)} + \Delta\mathbf{X}'^{(k)} \\ &= C'^{T}\,\mathbf{Y}' + \left(I_m - \frac{C'^{T}\,C'}{\left|C'\mathbf{X}'^{(k-1)}\right|}\right)\mathbf{X}'^{(k-1)},\end{aligned} \tag{9}$$

where I_m is an m by m unit diagonal matrix.

2.3 Convergence condition

Convergence of the iterative scheme Eq.(9) should be examined by considering a state transition matrix S from the solution vectors $\mathbf{X}^{(k-1)}$ to $\mathbf{X}^{(k)}$ in Eq.(9):

$$S = I_m - \frac{C'^{T}\,C'}{\left|C'\mathbf{X}'^{(k-1)}\right|}. \tag{10}$$

When the maximum eigen value of S is less than 1, the solution converges to an exact solution vector. However, the state transition matrix S in Eq. (10) is not a constant matrix but a function of the solution vector $\mathbf{X}'^{(k-1)}$. This means that the convergence depends on the solution vector $\mathbf{X}'^{(k-1)}$. The characteristic values of a unit square matrix are the multiple roots of 1. The convergence condition of the problem is described by:

$$\begin{aligned}&\left|I_m\right| \geq \left|S\right|, \\ &\left|C'\mathbf{X}'^{(k-1)}\right|\left|I_m\right| \geq \left\|C'\mathbf{X}'^{(k-1)}\right|I_m - C'^{T}\,C'\right|.\end{aligned} \tag{11}$$

In Eq.(11), all of the diagonal elements in the matrix $C'^{T}\,C'$ are 1, and the other elements of this matrix are always less than 1. Thereby, the condition of Eq.(11) is always held. This means that Eq.(9) gives an absolutely stable iterative solution.

3 INVERSE PROBLEM OF THE MAGNETIC FIELDS

3.1 System of equations

A relationship between current and magnetic field is given as a solution of Maxwell's equation. Applying a discretized integral equation method using a Green's function to the Maxwell's equation leads to a system of equations similar to Eq.(1). In a magnetic system, the vectors **Y**, **X** are the nth order magnetic field and the m[th] order current vectors, respectively. The n by m rectangular system matrix C is composed of the spatial derivatives of Green's function [2]. Major currents in electronic devices are flowing on the printed circuit boards, but the magnetic fields are spreading over a surrounding space. This means that a region containing the currents can be easily accessed whereas only a limited region containing magnetic fields is accessible. To access the region containing the currents, it is essentially required to destruct or decompose the devices. On the other hand, magnetic fields can be easily measured without destroying or decomposing the devices but only locally. Thereby, Eq.(1) has to be solved under the condition of $m>n$ in order to visualize the current distributions.

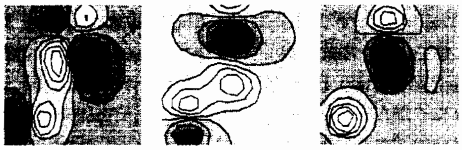
3.2 Validity of checking method of the solution

This paper proposes one of the methodologies to check up the validity of the GVSPM solutions. A key idea is to extract the common dominant parts among the independently evaluated GVSPM solutions. Each of the solutions is evaluated from the orthogonal x-, y- and z-components of the measured magnetic field vectors. Three kinds of current vector distributions are evaluated by means of the GVSPM.

The maximum magnitudes of these three current vector distributions are normalized to 1. To extract the common dominant current vector distributions, each of the current vector components was convoluted without using any threshold operation. This yields not only the reliable noise-free current distributions, but also reveals a validity of the solution.

3.3 Simulation

Fig.1 shows the magnetic field distributions, measured at a parallel surface, 1cm above a 10 by 10 cm printed circuit board. The three orthogonal components of the magnetic field were measured at 16 by 16 equi-spaced points. A 32×32 loop current distribution model was employed for the target printed circuit board so that the number of unknowns was $m=32\times32=1024$ [1,2]. Thus, the problem was to compute the 1024 loop currents from the 256 magnetic fields. After 100 iterations applying Eq.(9), the current vector distributions were computed.



a) x- b) y- c) z-

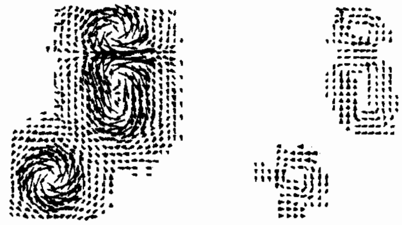
Figure 1 Measured magnetic field distributions at 16 by 16 equi-spaced points.

Fig.2 shows each of the independently evaluated current vector distributions normalized to 1. In order to extract the common current vector distributions each of the current vector components was convoluted. This convolution makes it possible to extract the only dominant current vector distributions without using any threshold operation.

a) x- b) y- c) z-

Figure 2 Current vector distributions evaluated from magnetic fields.

Fig.3 shows the convoluted current vector distribution together with the exact one. By observing the results in Fig.3, this methodology yields not only the reliable noise-free current vector distributions, but also reveals a validity of the GVSPM solution.



a) b)

Figure 3 Current vector distributions:
a) exact b) convoluted

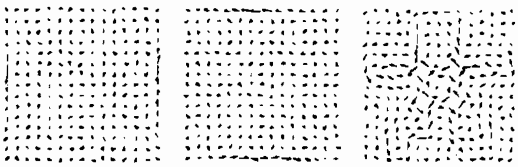
3.4 Experimental verification

Fig.4 shows the magnetic field distributions, which have been measured at a parallel surface 0.9 cm above a 10 by 10 cm target surface. The magnetic field components were measured at 10 by 10 equi-spaced points. Thereby, the entire number of magnetic field measured points was $10\times10=100$ for each of the x-, y- and z-components.

a) x- b) y- c) z-

Figure 4 Measured magnetic field distributions at 10 by 10 equi-spaced points.

A 16×16 loop current distribution model was employed for the target printed circuit board so that the number of unknowns for each component was $m=16\times16=256$. Thus, the problem was to compute the 256 loop currents from the 100 magnetic fields. After 300 iterations applying Eq.(9), the current vector distributions were computed

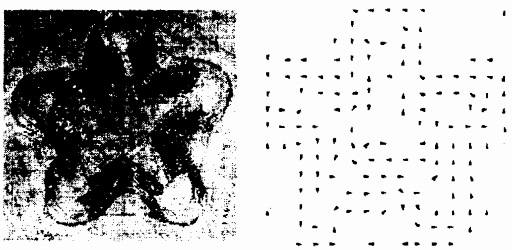


a) x- b) y- c) z-

Figure 5 Current vector distributions evaluated from magnetic fields.

Fig.5 shows each of the independently evaluated current vector distributions normalized to 1.

The common current vector distributions were extracted by the convolution of each of the current vector components. Fig.6 shows the current carrying coil with convoluted current vector distributions.



(a) (b)

Figure 6 (a) The current carrying coil
(b) The convoluted current vector distributions.

4 CONCLUSIONS

This paper has proposed one of the methodologies to check up the validity of GVSPM solutions. It yields reliable noise-free current distributions and reveals a validity of the GVSPM solution.

The visualization of the noise-free current vector distributions on the printed circuit boards has been successful. Thus, the methodology provides a deterministic tool for the nondestructive testing and also to the electromagnetic compatibility checking of modern electronic devices.

5 REFERENCES

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