A REPRESENTATION OF MAGNETIC AFTEREFFECT

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Abstract - A chua type magnetization model is presented and closely examined to the other models. As a result, it is revealed that a Chua type model is closely related with the Preisach type model and gives the Rayleigh relationships in the weakly magnetized region. Moreover, it is shown that the magnetization processes are essentially accompanying the magnetic aftereffects.

INTRODUCTION

Chua and Stromsmoe have worked out a lumped circuit model for nonlinear inductors exhibiting hysteresis loops [1]. Their model is based on the purely phenomenological behaivior of ferromagnetic materials, nevertheless their model exhibits many important hysteretic properties commonly observed in practice.

A Chua type model has been generalized to calculate the quasi-three dimensional magnetic fields in the magnetic devices [2]. Furthermore, it has been reported that a Chua type model is closely related with the Preisach type model [3], also its parameters may be determined by Fourier series expansion of the field intensity under the sinusoidal time varying magnetic flux density [4].

In the present paper, one of the Chua type models is derived from the ideal magnetization curve and revesisible permeability whose properties are not affected by the hysteretic properties. Also, it is shown that a combination of this model and Preisach type model yields the typical magnetization properties in the Rayleigh region. Furthermore,various aftereffects commonly observed in practice are reproduced by this Chua type model.

THE MAGNETIZATION MODEL

Chua Type Model

Since the solutions of magnetization model exhibit various magnetization properties such as the hysteresis, saturation and minor loops, then the model itself must be composed of the parameters not affected by hysteresis. One of the properties not affected by hysteresis is an ideal or anhysteretic magnetization curve which is obtained by first applying the superposed static and alternating fields, and then reducing the alternating field to zero and observing the flux density. This ideal magnetization curve may be represented by

$$H = (1/\mu)B, \tag{1}$$

where H, B and μ are the field intensity, flux density and permeability, respectively.

The other property not affected by hysteresis is the reversible permeability defined by

$$\mu_r = \Delta B/\Delta H, \tag{2}$$

where ΔB and ΔH are respectively the infinitesimally small alternating flux density and field intensity accompanying with the measurement process of ideal magnetization curve. Fig. 1 shows a relationship between the ideal magnetization curve and associated reversible permeability. From an experimental point of view, the magnetization is accomplished in essence through the time variation of flux density and field intensity. Thereby, (2) may be rewritten by

$$\partial H/\partial t = (1/\mu_r)\partial B/\partial t. \tag{3}$$

After introducing a hysteresis coefficient s [Ohm/m] into (3), cosideration of total field intensity due to (1) and (2) yields a following relation:

$$H + (\mu_r/s)(\partial H/\partial t) = (1/\mu)B + (1/s)\partial B/\partial t, \tag{4}$$

or

$$H = (1/\mu)B + (1/s)[(\partial B/\partial t)-\mu_r(\partial H/\partial t)]. \tag{5}$$

Obviously, (4) or (5) is one of the Chua type models [2-4].

Preisach Type model

According to the [5], the reversing point field intensity H_n and applied field intensity H_p are defined as shown in Fig. 2. By considering Fig. 2, it is obvious that B-H trajectory takes different paths depending on the reversing field intensity H_n. Thereby, the flux density B is represented as a function of applied field intensity H_p as well as reversing point field intensity H_n,

$$B = B(H_p,H_n). \tag{6}$$

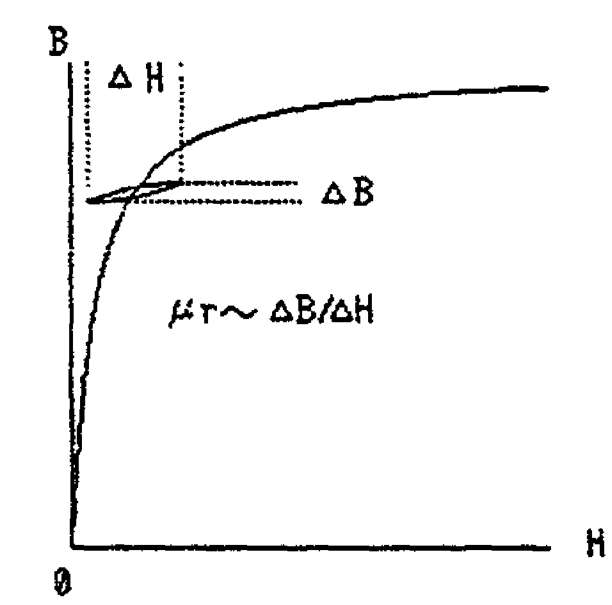

Fig. 1. Ideal magnetization curve and accompanying reversible permeability $\mu_r=\Delta B/\Delta H$.

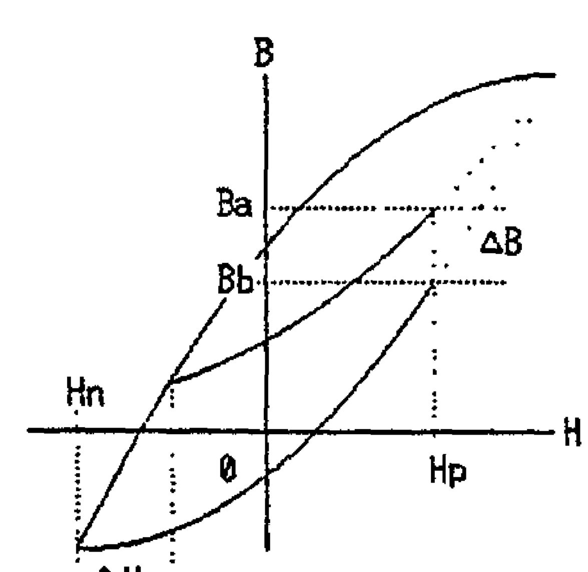

Fig. 2. Nonsymmetrical hysteresis loops for the derivation of Preisach model.

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Moreover, by considering a saturation point of flux density on the nonsymmetrical hysteresis loop shown in Fig. 2, it is revealed that the B-H trajectories take different paths according to each of the reversing points of field intensity but always coincide at the saturation point of flux density. Therefore, the rate of change of slope $\partial B/\partial H_p$ with the reversing point field intensity H_n takes non-zero value in the region $|B|<B_m$, where B_m is the saturation flux density. This relationship gives the definition of Preisach's function Ψ as

$$\Psi = \partial^2 B(H_p,H_n)/\partial H_n\, \partial H_p. \tag{7}$$

In order to find a relationship between the Chua and Preisach type models, application of (5) to the states of Fig. 2 gives

$$H_p = (1/\mu)B_a + (1/s)[(\partial B_a/\partial t)-\mu_r(\partial H_p/\partial t)], \tag{8}$$

$$H_p = (1/\mu)B_b + (1/s)[(\partial B_b/\partial t)-\mu_r(\partial H_p/\partial t)], \tag{9}$$

where the field intensity ΔH_n in Fig.2 is so small that the permeabilitis μ,μ_r and hysteresis coefficient s may be assumed to be constants. By subtracting (8) from (9) and rearranging, it is possible to obtain

$$\Delta B/\mu = (1/\mu)(B_a-B_b) = (1/s)[(\partial B_b/\partial t)-(\partial B_a/\partial t)]$$

$$= (1/s)[(\partial B_b/\partial H_p)-(\partial B_a/\partial H_p)]\partial H_p/\partial t. \tag{10}$$

Further rearrangement of (10) yields

$$s/(\partial H_p/\partial t) = (\mu/\Delta B)[(\partial B_b/\partial H_p)-(\partial B_a/\partial H_p)]. \tag{11}$$

In Fig. 2, if the limit of ΔH_n goes to zero, then $\Delta B/\mu$ term in (11) is simultaneously reduced to zero. Thus, an assumption of $\Delta H_n=\Delta B/\mu$ leads to

$$\lim_{\Delta H_n \to 0} (\mu/\Delta B)[(\partial B_b/\partial H_p)-(\partial B_a/\partial H_p)]$$

$$= \partial^2 B/\partial H_n\, \partial H_p. \tag{12}$$

From (7), (11) and (12), a relationship between the hysteresis coefficient s and Preisach's function Ψ is obtained as

$$s = \Psi(\partial H/\partial t). \tag{13}$$

Substituting (13) into (4) yields a modified Chua type model:

$$H + (\mu_r/\Psi) = (1/\mu)B + (1/\Psi)\partial B/\partial H. \tag{14}$$

THE MAGNETIZATION PROPERTIES

Initial Portion of Curve

When the parameters μ, μ_r and Ψ in (14) are assumed to take the constant values in the weakly magnetized region known as the Rayleigh region, then (14) gives

$$B = \mu(H_n+H_p)+(\mu^2/\Psi)[1-(\mu_r/\mu)][\exp\{-(\Psi/\mu)(H_p+H_n)\}-1]$$

$$- B_n\exp\{-(\Psi/\mu)(H_p+H_n)\}, \tag{15}$$

where H_p, H_n and B_n are the applied field intensity, reversing field intensity and reversing point flux density, respectively. The field intensities H_p,H_n are so small that the following assumptions are possible:

$$\exp\{-(\Psi/\mu)(H_p+H_n)\} \approx 1-(\Psi/\mu)(H_n+H_p)$$

$$+ (1/2)\{(\Psi/\mu)(H_n+H_p)\}^2. \tag{16}$$

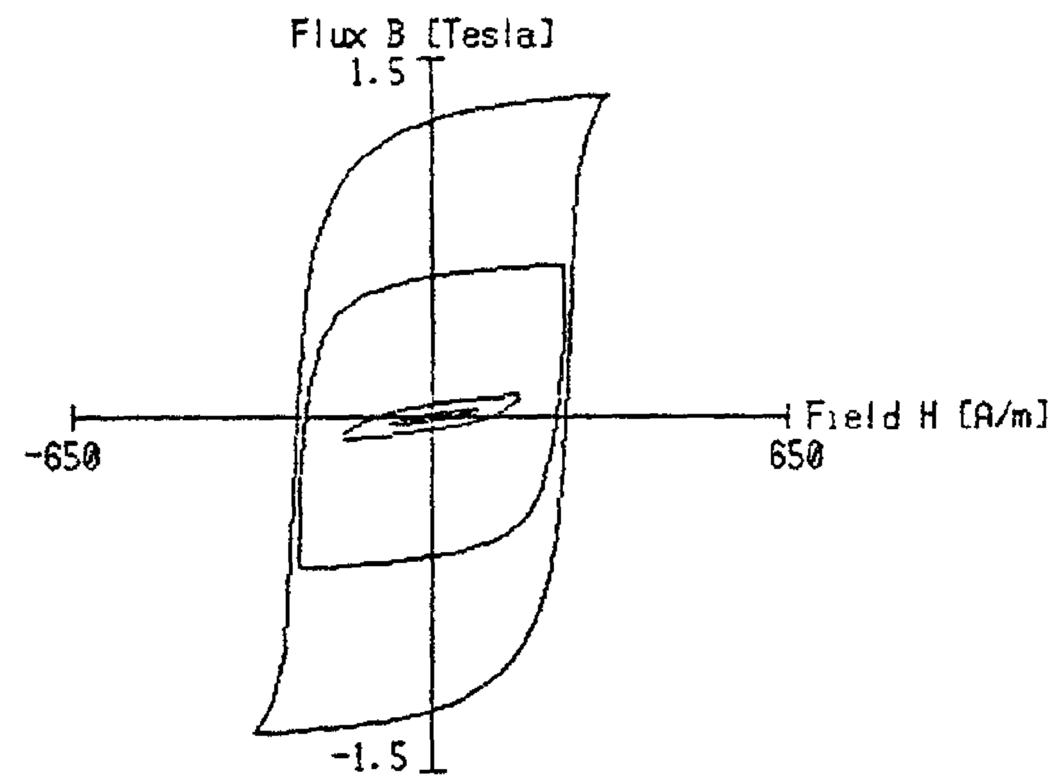


Fig. 3. A family of hysteresis loops obtained by (4) using the curves in Fig. 4 and μ_r=.000273[H/m]

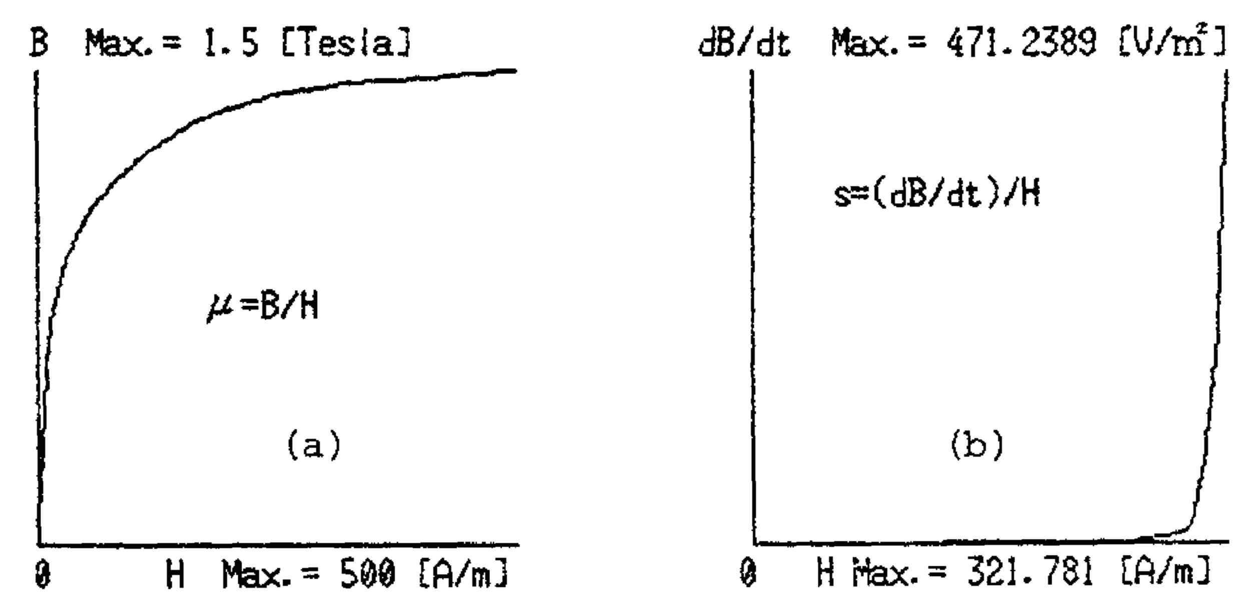
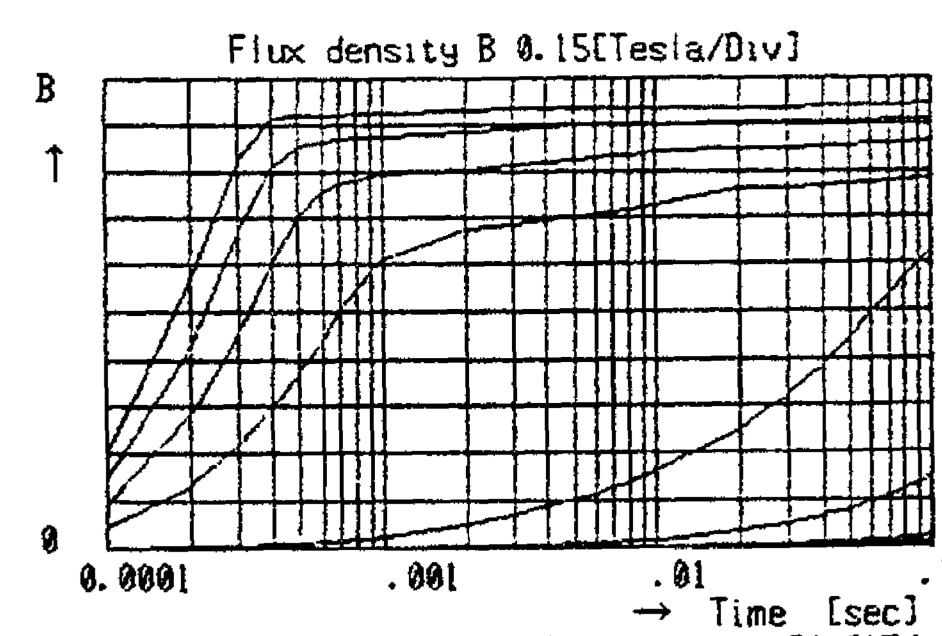
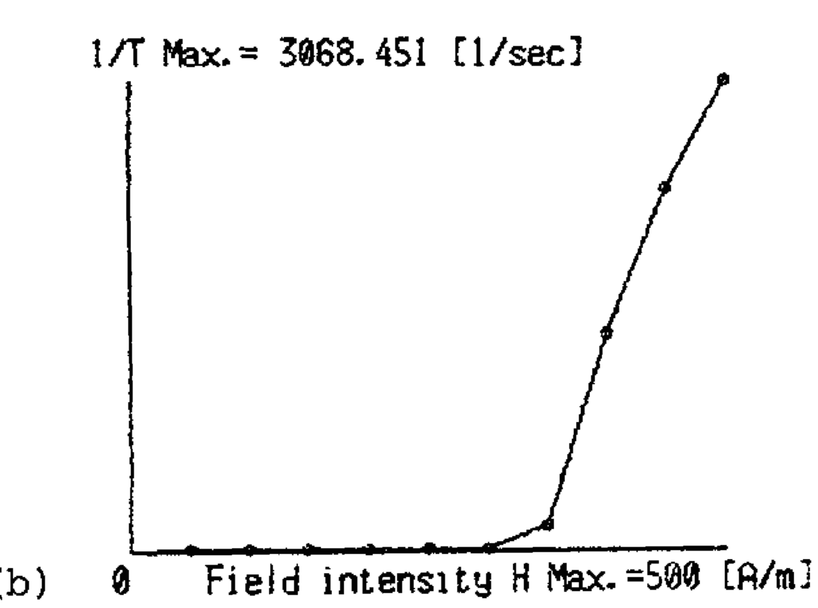


Fig. 4. (a) Ideal magnetization curve for μ[=B/H], and (b) dB/dH vs. H curve for s[=(dB/dt)/H].



(a) Filed intensity H 50 ~ 500 , Step = 50 [AT/m]



(b)

Fig. 5. (a) A family of aftereffect obtained by applying the step field intensities, and (b) time required to reach the 90 % of each final magnetization.

When (16) is substituted into (15) and H_n, B_n are set to be zero, then (15) is reduced to

$$B = \mu_r H_p + (1/2)\Psi H_p^2[1-(\mu_r/\mu)]$$

$$\approx \mu_r H_p + (1/2)\Psi H_p^2, \qquad (17)$$

where $\mu >> \mu_r$ is assumed. (17) is obviously Rayleigh's relation [6]. Hence, it is revealed that the Preisach's function Ψ corresponds to the Rayleigh's constant.

Consideration of symmetrical loop condition gives the reversing point flux density B_n as

$$B_n = \mu H_n + [\mu H_n - (\mu^2/\Psi) + (\mu\mu_r/\Psi)]\tanh[(\Psi/\mu)H_n]. \qquad (18)$$

When (18) is approximated as those of (16) and the approximations of (16),(18) are substituted into (15), then (15) is reduced to

$$B = (\mu_r + \Psi H_n)H_p + (\Psi/2)(H_p^2 - H_n^2), \qquad (19)$$

where over the third order terms are neglected, also $\mu >> \mu_r$ is assumed. (19) is known as a lower branch of Rayleigh loops. Thus, a modified Chua type model (14) yields the Rayleigh's relationships in the weakly magnetizated region.

As shown in Fig. 3, the Rayleigh loops are obviously observed in the weakly magnetized region. A family of hysteresis loops in Fig. 3 were obtained by means of (4), where $\mu_r = 2.73\times10^{-4}$ [H/m] and the other parameters μ, s were evaluated from Fig. 4.

Aftereffect

The magnetization of (15) is essentially accompanying the aftereffect, because (15) is rewritten as a solution of (4) by

$$B = \mu H_m(t/T) + (H_m/T)(\mu/s)(\mu_r-\mu)[1-\exp\{-(s/\mu)t\}], \qquad (20)$$

where H_n, B_n in (15) are set to be zero; H_m and T are the maximum field intensity and time required to reach H_m, respectively. (20) means that the magnetization process in the weakly magnetized region is carried out accompanying with the aftereffect described by a single exponential function.

Fig.5 shows the aftereffect when the the step fields are applied. The results of Fig. 5 were calculated by means of (4) using the similar parameters of Fig. 3. Observation of the results in Fig. 5 reveals that the magnetization is greately stimulated when the applied step field intensities become to the larger than a threshold value.

Aftereffect is observed when the applied field intensity is removed. Fig. 6 shows the time variations of flux density after the removal of field intensities. Also, the results of Fig. 6 were obtained by means of (4) using the similar parameters of Fig.3. The observed aftereffect shown in Fig. 6 is known as a Richter type aftereffect [6].

CONCLUSION

As shown above, we have derived a Chua type model for representing the magnetization properties, and closely examined the relationships to the other models. As a result, it has been clarifed that the Preisach' function is closely related with the hysteresis coefficient, and Rayleigh's relationships are included in a Chua type model. Furthermore, a Chua type model has suggested that all of the magnetization processes are accomplished in essence through the magnetic aftereffect.

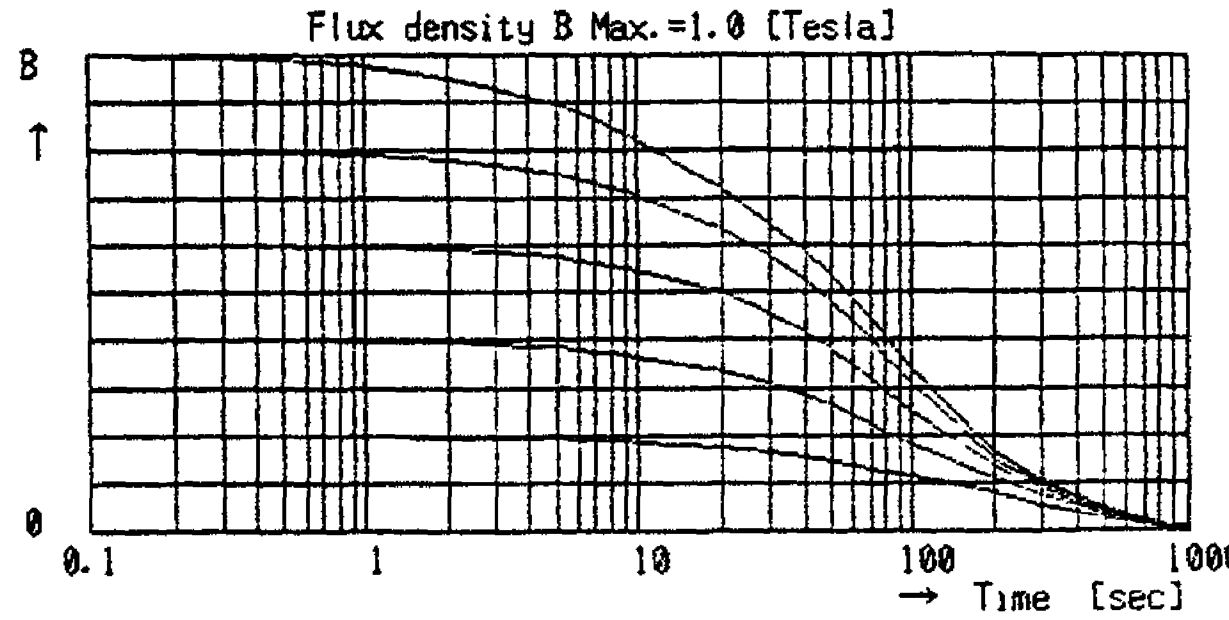


Fig. 6. A family of typical Richter type aftereffects obtained by the removal of field intensities.

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