FINITE ELEMENT SOLUTION OF UNBOUNDED MAGNETIC FIELD PROBLEM CONTAINING FERROMAGNETIC MATERIALS

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Abstract: A great interest has been focused on the strategic dual image method for obtaining the finite element solution of unbounded magnetic field problem [1]. Even though the strategic dual image method can be applied to any complex magnetic field distributions with unbounded boundary, a difficulty of the generalization to the magnetic field problem containing ferromagnetic materials has been pointed out.

In this paper, we show that this difficulty can be removed by means of the magnetization vector. As a result, it is shown that our strategic dual image method permits the finite element solution of unbounded magnetic field problems containing ferromagnetic materials.

INTRODUCTION

The finite element and difference methods are being widely used to solve the various engineering field problems. However, because of their essential feature, it is difficult to obtain the solution of the unbounded field problems. To overcome this difficulty, we have previously proposed a new method which is based on the method of electrical image [1,2]. This new method is called the strategic dual image method (SDI). Even though, our SDI method is capable of any unbounded field problems,a difficulty of the application to the unbounded field problems containing ferromagnetic materials has been pointed out.

In this paper, it is revealed that this difficulty can be removed by utilizing the $B=\mu_0(H+M)$ formulation instead of $B=\mu H$, where B, H, M, μ_0 and μ are respectively the flux density, field intensity, magnetization, permeability of air and permeability of ferromagnetic material. As a result, it is shown that our SDI method can be applicable to obtain the finite element solution of unbounded saturable magnetic field problems.

THE STRATEGIC DUAL IMAGE METHOD

Basic field equation

In the ferromagnetic materials, a magnetization vector **M** is related with the field H as

$$\mathbf{M}=\chi_m \mathrm{H}, \tag{1}$$

where χ_m is called the magnetic susceptibility of the material. Also,the magnetization flux density B is given by

$$\mathrm{B}=\mu_0 \ (\mathrm{H}+\mathbf{M}) , \tag{2}$$

where μ_0 is the permeability of air $4\pi\times10^{-7}$[H/m]. By means of (1), it is possible to modify (2) as

$$\mathrm{B}=\mu_0(1+\chi_m)\mathrm{H}=\mu_0\mu_r\mathrm{H}=\mu\mathrm{H}, \tag{3}$$

where μ, μ_r are respectively called the permeability and relative permeability of the material. When we employ a vector potential **A** and assume the Coulomb gauge, then (2) leads to the following governing equation:

$$(1/\mu_0)\nabla^2 \mathbf{A}=-\mathbf{J}_s-\mathbf{J}_i \, . \tag{4}$$

where $\mathbf{J}_s$ and $\mathbf{J}_i$ are respectively the source current density and magnetizing current density.

Similarly, the following governing equation can be derived from (3) as

$$(1/\mu)\nabla^2 \mathbf{A}=-\mathbf{J}_s \, , \tag{5}$$

where a constant permeability μ has been assumed, because most of the numerical methods always utilize a piecewise linear permeability. Combination (4) with (5) gives the magnetizing current density $\mathbf{J}_i$, viz.,

$$-\mathbf{J}_i=(1/\mu_0)\nabla^2 \mathbf{A}-(1/\mu)\nabla^2 \mathbf{A}. \tag{6}$$

Thus the effect of ferromagnetization is represented in terms of the magnetization current density $\mathbf{J}_i$.

Infinite boundary

In most engineering problems, the net magnetic field source becomes zero, so the net magnetic field source is assumed to be zero in the problem region. Furthermore as shown in Fig.1, it is assumed that the problem region is enclosed by a boundary located at an infinitely long distance from the problem region. By considering this infinite boundary, it is obvious that the boundary conditions at the infinite boundary are

$$\mathrm{B}_n=0, \tag{7}$$

$$\mathrm{H}_t=0, \tag{8}$$

where the flux density B_n and field intensity H_t are, respectively, the normal and tangential components to the infinite boundary as illustrated in Fig.1. The boundary conditions (7) and (8) suggest that the unbounded field is composed of the divergent field B_n and rotational field H_t. This means that the magnetic field source can be regarded

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as a rotational field source, i.e., current i, or divergent field source, i.e., magnetic charge m.

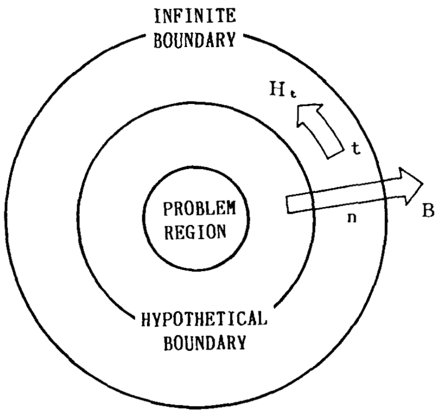


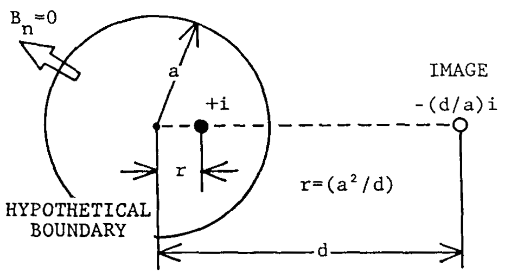
FIG.1. Modeling of unbounded field problems.

Rotational field component

When we impose $-(d/a)\ i$ as shown in Fig.2(a), then the normal component of flux density **B** to the spherical surface is suppressed by the image current $-(d/a)\ i$ so that the remaining field on the spherical surface is the tangential field intensity **H**. Thus the rotational field component H_t which becomes zero at the infinite boundary can be evaluated. The magnitude of image $-(d/a)\ i$ depends on a position of field source i in the sphere so that the following condition

$$a\sum_{p=1}^{q}(i_p/r_p)=0, \tag{9}$$

must be satisfied to reduce the zero net image for all the currents in the problem region. In (9), a is sphere radius; r_p $(=a^2/d_p)$ is the distance from the center of the sphere to the current i_p; and q is the number of sources. Equation (9) means that the vector potential **A** must be reduced to zero at the center and the surface of the sphere.



(a) The rotational field source image $-(d/a)\ i$ and hypothetical spherical boundary. The normal components of flux density at the spherical surface become zero.

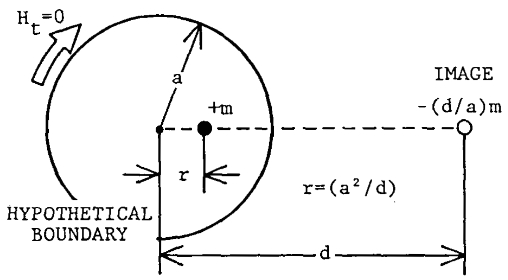
FIG.2. Strategic dual images.

Divergent field component

Let's consider one of the magnetic charges m in the problem regions. When we impose an image $-(d/a)\ m$ at a position shown in Fig.2(b), then the tangential component of the field intensity **H** becomes zero at an arbitrary point on the sphere enclosing the field source m. This means that the tangential component of the field intensity **H** to the spherical surface is suppressed by the image $-(d/a)\ m$ so that the remaining field on the spherical surface is the normal component of the flux density **B**. Thus, the divergent field component B_n which becomes zero at the infinite boundary can be evaluated. Similarly to (9), the following condition

$$a\sum_{p=1}^{q}(m_p/r_p)=0, \tag{10}$$

must be satisfied to reduce the zero net image for all the magnetic charges in the problem region. The magnetic field source, in this case, is regarded as the magnetic charge m instead of the current i so that the condition (9) still holds. This means that the vector potential **A** must be reduced to zero at the center of the sphere. Also, the boundary condition at an arbitrary point on the spherical surface is represented by the symmetrical condition $\partial A/\partial n=0$, where n denotes the normal direction to the spherical surface.



(b) The divergent field source image $-(d/a)\ m$ and hypothetical spherical boundary. The tangential components of field intensity at the spherical surface become zero.

FIG.2. Strategic dual images.

Unbounded field

The unbounded field is composed of the rotational and divergent field components. The rotational and divergent fields are respectively evaluated by setting the boundary condition $\mathbf{A}=0$ and $\partial A/\partial n=0$ at the spherical surface. Further, because of (9), the condition $\mathbf{A}=0$ must be imposed at the center of the sphere enclosing all the field source. Thus, the unbounded field can be obtained by a summation of these two fields. It must be noted that an unbounded field has two field sources (rotational and divergent field sources) so that the original field is obtained by dividing the total field by two, that is

$$X = (1/2)\,(X_s + X_r)\,, \tag{11}$$

where X, X_s and X_r are the unbounded, divergent and rotational field solution vectors, respectively.

Implementation

The magnetic field is represented by the curl of vector potential A so that the vectors representing the rotational and divergent field components are, respectively, calculated by setting the boundary conditions $A = 0$ and $\partial A / \partial n = 0$ at the spherical surface of the hypothetical boundary. Furthermore, because of (9), $A = 0$ must be satisfied at the center of the spherical surface.

When the rotational field solution vector X_r and the divergent field solution vector X_s are obtained after solving each of the systems, the unbounded field solution vector X is obtained from (11). In (11), it is required to use the rotational and divergent field solutions. But it is possible to show that the SDI method does not require double computations for rotational and divergent field solutions. This is based on the following fact:

$$X = (1/2)\,X_s \tag{12}$$

is established at the hypothetical boundary because the other solution vector X_r is always zero.

On the other hand, the normal derivative $\partial X / \partial n$ at the hypothetical boundary becomes

$$\partial X / \partial n = (1/2)\,\partial X_r / \partial n\,, \tag{13}$$

because the normal derivative $\partial X_s / \partial n$ of the solution X_s is zero. Therefore, the SDI method can be implemented by using either boundary condition (12) or (13) not requiring the double computations for X_s and X_r in (11). According to (4) and (6), it is obvious that the solution vector X essentially becomes a function of X. This means that the solution vector X should be iteratively evaluated in the following manner[3]

$$X^{(k+1)} = f\,[X^{(k)}]\,, \tag{14}$$

where the superscripts (k+1) and (k) denote the (k+1) and (k)th iterations, respectively.

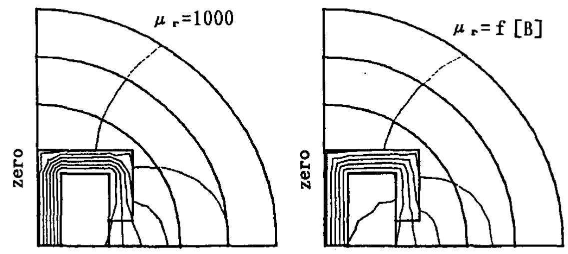
An example

In this paper, the strategic dual image method has been formulated in three dimensions. However, because of its simplicity, the two-dimensional static magnetic field problems are far more preferable as an initial test example.

The discretization was carried out by means of first-order triangular finite elements. Because of a symmetrical property of the problem region, the computations were carried out on the quarter portion of an entire problem region.

Figure 3(a) shows a magnetic field distribution in an 'E' core magnet. The solutions were obtained using the hypothetical boundaries with different radii. Nevertheless, the solutions almost coincided with each other. This means that our SDI method gives a unique solution for the unbounded magnetic field containing ferromagnetic materials.

Figure 3(b) shows a magnetic field distribution in an 'E' core magnet taking into account the magnetic saturation. A comparison of Fig.3(a) with 3(b) shows a notable difference in the vicinity of the air gap.



(a) Linear solutions (b) Non-linear solutions

FIG.3. FEM solutions in the 'E' core magnet,
—— 3[m] radius solution,
—·— 4[m] radius solution,
---- 5[m] radius solution.

CONCLUTION

We have shown above that the strategic dual image method is applicable to unbounded magnetic field problems containing ferromagnetic materials. As a result, it may be concluded that the finite element solutions of any unbounded field problems can be obtained by the SDI method using an extremely simple procedure.

REFERENCES

[1] Y.Saito., et al, "Finite element solution of open boundary magnetic field problems," IEEE Trans., Magn., Vol.MAG-23, 3569 (1987).

[2] Y.Saito., et al, "The strategic dual image method an extremely simple procedure for open field problems," Journal of Applied Physics, Vol.63, No.8, 3366 (1988).

[3] Y.Saito., et al, "Strategic dual image method for open boundary electromagnetic field calculation," Papers of Technical Meeting on Magnetics, IEE Japan, SHINSHU, April (1988).