A representation of magnetization characteristics and its application to the ferroresonance circuits

Y. Saito, M. Namiki, and S. Hayano
College of Engineering, Hosei University, 3-7-2 Kajinocho, Koganei, Tokyo 184, Japan

Previously, a Chua-type model representing the magnetization characteristics of ferromagnetic materials was presented for computational analysis and design of magnetic devices. Typical magnetization characteristics, such as saturation, hysteresis, and aftereffect, have been reproduced by the model. In this paper, our Chua-type model is derived by means of the barlike domain walls model. As a concrete example, this model is applied to the computations of ferroresonance circuits and is compared with those of the experimental results. Agreement to a considerable extent confirms the validity of this model.

I. INTRODUCTION

With the development of modern high-speed digital computers, numerical methods became available to calculate the magnetic fields in magnetic devices. In order to calculate the accurate magnetic fields in electromagnetic devices, it is required to work out a model representing the magnetization characteristics of ferromagnetic materials.

Ferromagnetic materials exhibit various characteristics, e.g., saturation, hysteresis, aftereffect, frequency dependence, magnetostriction, mechanical stress dependence, and temperature dependence.[1] Among these magnetization characteristics, it must be taken into account the saturation, hysteresis, aftereffect, and frequency dependence for the computational analysis and design of electromagnetic devices.

The models representing magnetization characteristics may be classified into two-major categories. One is a Preisach-type model, which assumes that each of the magnetic domains has a rectangular hysteresis loop, and interaction between domains can be introduced by examining local field acting on domains.[2] Even though the Preisach-type model is based on such simple assumptions, it gives valuable results that are in agreement with experimental results.[3] However, there is an instable problem for which the Preisach's function takes a different value depending on the previous path in the magnetization processes.[4] This Preisach-type model has been extended to be used for anisotropic and dynamic magnetization processes by Mayergoyz.[5–7] The other is a Chua-type model, which has been derived on the purely phenomenological behaviors of ferromagnetic materials. The key idea of the Chua-type model is that a trajectory of flux linkage against current is uniquely determined by the last point at which the time derivative of flux linkage changes sign.[8] Also, this Chua-type model could be derived by the Fourier series expansion of the field intensity under the sinusoidal time varying flux density.[9] The Chua-type model exhibits many important magnetization properties, e.g., the presence of minor loops and an increase in area of the loop with frequency. Subsequently, a Chua-type model has been derived by considering the static and dynamic behaviors of magnetization. This specific Chua-type model is capable of representing Rayleigh's relationship, aftereffect, and iron loss.[10,11] Furthermore, it has been pointed out that one of the parameters of the Chua-type model is closely related with the movement of domain walls, and it has been confirmed that the frequency dependence, minor loop, and transient magnetization are satisfactorily reproduced by the Chua-type model.[12,13]

In this paper, the Chua-type model is derived by means of the barlike domain walls model. As a practical example of computational analysis based on the Chua-type model, the behaviors of a series ferroresonance circuit are computed and compared with those of the experimental results. Agreement to a considerable extent confirms the usefulness of the Chua-type model.

II. THE CHUA-TYPE MAGNETIZATION MODEL

As shown in Fig. 1, we consider a barlike magnetic domain walls model; then the following relationship can be established:

$$B = \mu_0 H + nB_s \quad (1a)$$

$$= \mu_0 [1 + (nB_s/\mu_0 H)] H \quad (1b)$$

$$= \mu H, \quad (1c)$$

where B, B_s, H, n, μ_0, and μ are the flux density, saturation flux density of each domain, field intensity, number of domains in accordance with the direction of external field H, permeability of air, and permeability of material, respectively. When relationship (1) is measured by first applying the superposed static and alternating fields, then reducing the alternating field to zero and observing the flux density, obtaining B-H relation becomes a unique characteristic which isnot affected by the past magnetization histories. This is known as an ideal or anhysteretic magnetization curve. In the other words, relationship (1) represents a static magnetization characteristic corresponding to an each of the do-

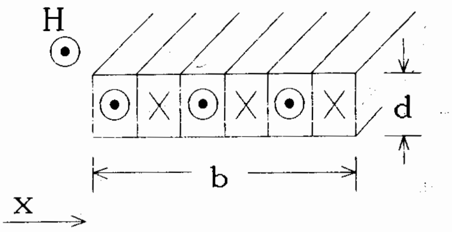

FIG. 1. Barlike domain walls model.

 0021-8979/90/094738-03$03.00

main situations when a unique single valued relation $B = \mu H$ can be held.

Differentiating (1) with time t, it is possible to obtain the following relation:

$$\frac{dB}{dt} = \mu_0 \frac{dH}{dt} + B_s \frac{dn}{dt} \quad (2a)$$

$$= \left(\mu_0 + B_s \frac{\partial n}{\partial H}\right)\frac{dH}{dt} + B_s \frac{\partial n}{\partial x}\frac{dx}{dt} \quad (2b)$$

$$= \mu_r \frac{dH}{dt} + B_s \frac{\partial n}{\partial x} v, \quad (2c)$$

where $v = (dx/dt)$ denotes a velocity of magnetic domain walls motion. Equation (2) means that the magnetization of ferromagnetic materials accompanies the time variations of field intensity dH/dt and physical domain movement dx/dt. In the other words, the induced voltage per unit area dB/dt is composed of two components: the transformer- and velocity-induced voltages. Introducing a hysteresis coefficient $s(\Omega/m)$, a magnetic field due to the magnetic domain movement is given by[9-13]

$$\left(\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right)\Big/ s = \left(B_s \frac{\partial n}{\partial x} v\right)\Big/ s. \quad (3)$$

Consideration of a total field intensity due to the static (1) and dynamic (3) magnetizations yields the following relation:

$$H = \frac{1}{\mu} B + \left(\frac{dB}{dt} - \mu_r \frac{dH}{dt}\right)\frac{1}{s} \quad (4a)$$

$$= \frac{1}{\mu} B + \left(B_s \frac{\partial n}{\partial x} v\right)\frac{1}{s}, \quad (4b)$$

or

$$H + \frac{\mu_r}{s}\frac{dH}{dt} = \frac{1}{\mu} B + \frac{1}{s}\frac{dB}{dt}, \quad (4c)$$

where the parameters μ, μ_r, and s can be represented as the single valued functions of B, dB/dt, and dH/dt. The concrete examples of these papameters are shown in Fig. 2. Equation (4) is a Chua-type model.

By means of (2c) and (4b), an iron loss per unit volume is given from

$$H\frac{dB}{dt} = \frac{1}{\mu} B \frac{dB}{dt} + B_s \frac{\partial n}{\partial x}\frac{v}{s}\left(\mu_r \frac{dH}{dt} + B_s \frac{\partial n}{\partial x} v\right). \quad (5)$$

According to Refs. 10–13, a relationship between the hysteresis coefficient s in (4c) and Preisach's distribution function Ψ is given by

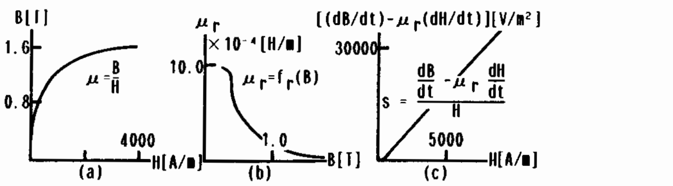

FIG. 2. (a) Anhysteretic magnetization curve, $\mu = B/H = \mu_0 + (nB_s/H)$; (b) reversible permeability, $\mu_r = f_r(B) = \mu_0 + B_s(\partial n/\partial H)$, where B is a bias flux density; (c) hysteresis coefficient, $s = [(dB/dt) - \mu_r(dH/dt)]/H = B_s(\partial n/\partial x)(dx/dt)/H$.

$$s = \Psi \frac{dH}{dt}. \quad (6)$$

Substituting (6) into (5) yields

$$H\frac{dB}{dt} = \frac{1}{\mu} B \frac{dB}{dt} + B_s \frac{\partial n}{\partial x}\frac{\mu_r}{\Psi} v + \left(B_s \frac{\partial n}{\partial x} v\right)^2 \frac{1}{s}, \quad (7)$$

where the first term on the right-hand side corresponds to a stored magnetic power; and the velocity v is obviously proportional to an exciting frequency $v \propto f$ so that the remaining second and third terms on the right-hand side, respectively, correspond to the hysteresis loss ($\propto f$) and eddy current loss ($\propto f^2$).

III. APPLICATION TO A SERIES FERRORESONANCE CIRCUIT

The circuit containing a nonlinear inductor leads to a nonlinear phenomena.[14,15] As an example of the nonlinear phenomena, we examined a R-L-C series ferroresonance circuit. Figure 3 shows a schematic diagram of the R-L-C series ferroresonance circuits. Application of the Chua-type model (4c) to the reactor shown in Fig. 3 gives

$$\int_0^D \left(H + \frac{\mu_r}{s}\frac{dH}{dt}\right) dl = \int_0^D \left(\frac{1}{\mu} B + \frac{1}{s}\frac{dB}{dt}\right) dl, \quad (8)$$

where D and dl are the mean length of flux path and infinitesimally small distance along the flux path D, respectively. With A denoting a cross-sectional area normal to the flux path, the right-hand term of (8) can be rewritten by

$$\int_0^D \left(\frac{1}{\mu} B + \frac{1}{s}\frac{dB}{dt}\right) dl = \frac{1}{L_i}\phi + \frac{1}{R_i}\frac{d\phi}{dt}, \quad (9)$$

where $\phi = AB$, $L_i = \mu A/D$, and $R_i = sA/D$. Also, the left-hand term of (8) can be rewritten in terms of the current i and number of turns of the coil N as

$$\int_0^D \left(H + \frac{\mu_r}{s}\frac{dH}{dt}\right) dl = Ni + N\frac{\mu_r}{s}\frac{di}{dt}. \quad (10)$$

The relationships among the flux ϕ, current i, electrical resistance r of coil, impressed voltage e, and terminal voltage e_c of capacitor C are given by

$$i = \frac{1}{r}\left(e - e_c - N\frac{d\phi}{dt}\right), \quad (11a)$$

$$\frac{di}{dt} = \frac{1}{r}\left(\frac{de}{dt} - \frac{de_c}{dt} - N\frac{d^2\phi}{dt^2}\right), \quad (11b)$$

and

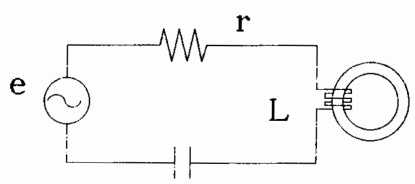

FIG. 3. Schematic diagram of a series ferroresonance circuit.

TABLE I. Various constants used in the calculations.

Area normal to the flux path	$A = 0.0001$ (m^2)
Mean length of the flux path	$D = 0.28$ (m)
Number of turns of coil	$N = 900$ (turn)
Electric resistance of coil	$r = 6.3(\Omega)$
Capacitance	$C = 100(\mu F)$

$$C\frac{de_c}{dt} = \frac{1}{r}\left(e - e_c - N\frac{d\phi}{dt}\right). \quad (11c)$$

By means of (8)–(11b), it is possible to derive the following relationship:

$$\frac{N}{r}\left(e + \frac{\mu_r}{s}\frac{de}{dt}\right) = \frac{N^2}{r}\frac{\mu_r}{s}\frac{d^2\phi}{dt^2} + \left(\frac{N^2}{r} + \frac{1}{R_i}\right)\frac{d\phi}{dt} + \frac{N}{r}\frac{\mu_r}{s}\frac{de_c}{dt} + \frac{1}{L_i}\phi + \frac{N}{r}e_c. \quad (12)$$

Equations (11c) and (12) were simultaneously solved for flux ϕ and voltage e_c by a conventional trapezoidal rule under the various exciting voltages e. The current i was evaluated from the flux ϕ and voltage e_c by (11a). Various constants used in the calculations are listed in Table I. Parameters μ, μ_r, and s of model (4) were obtained from the Fig. 2.

Figure 4 shows a computed ferroresonance phenomenon together with an experimental result when an amplitude modulated voltage e is impressed to the ferroresonance circuit.

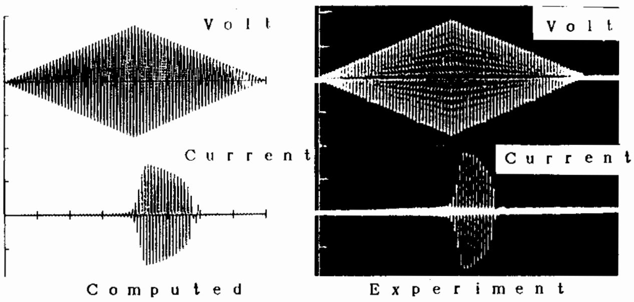

FIG. 4. Ferroresonance phenomenon caused by an amplitude modulated input voltage; time t: 0.1(s)/div; Voltage e: 50(V)/div; Current i: 10 (A)/div; frequency $f = 100$(Hz).

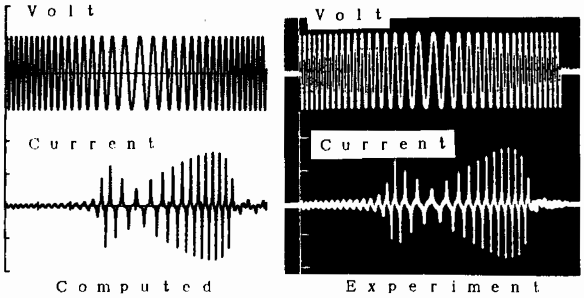

FIG. 5. Ferroresonance phenomenon caused by a frequency modulated input voltage; time t: 10(ms)/div; Voltage e: 20(V)/div; Current i: 2(A)/div; frequency f: 70→20→70 (Hz).

On the other side, when we impress a frequency modulated voltage e to the ferroresonance circuit, then a ferroresonance phenomenon is also observed, as shown in Fig. 5. Both of the results in Figs. 4 and 5 demonstrate the usefulness of our Chua-type model for the computational analysis and design of electromagnetic devices.

IV. CONCLUSION

As shown above, we have derived a Chua-type magnetization model based on the barlike domain walls model. Furthermore, this Chua-type model has been applied to the calculations of ferroresonance phenomenon. As a result, one of the versatilities of the Chua-type model has been demonstrated.

[1] R. M. Bozorth, *Ferromagnetism* (Van Nostrand, New York, 1951).
[2] F. Preisach, Z. Phys. **94**, 277 (1935).
[3] S. Sawamura and S. Iwasaki, IEEE Trans. Magn. **MAG-6**, 646 (1970).
[4] S. Shimano and N. Tsuya, Record of Discussion at Electrical Communication Lab., Tohoku University, **35**, 127 (1966).
[5] I. Mayergoyz, IEEE Trans. Magn. **MAG-22**, 603 (1986).
[6] I. Mayergoyz, IEEE Trans. Magn. **MAG-23**, 2638 (1987).
[7] I. Mayergoyz, IEEE Trans. Magn. **MAG-24**, 2925 (1988).
[8] L. O. Chua and K. A. Stromsmoe, IEEE Trans. Circuits Theor. **CT-17**, 564 (1970).
[9] Y. Saito, S. Hayano, H. Nakamura, Y. Kishino, and N. Tsuya, J. Magn. Magn. Mater. **54–57**, 1613 (1986).
[10] Y. Saito, S. Hayano, Y. Kishino, K. Fukushima, H. Nakamura, and N. Tsuya, IEEE Trans. Magn. **MAG-22**, 647 (1986).
[11] Y. Saito, K. Fukushima, S. Hayano, and N. Tsuya, IEEE Trans. Magn. **MAG-23**, 2227 (1987).
[12] Y. Saito, S. Hayano, and Y. Sakaki, J. Appl. Phys. **64**, 5684 (1988).
[13] Y. Saito, M. Namiki, S. Hayano, and N. Tsuya, IEEE. Trans. Magn. (to be published).
[14] J. G. Santesmases, J. Ayala, and H. Cachero, Proc. IEE **117**, 234 (1970).
[15] Y. Saito, H. Saotome, and T. Yamamura, Comp. Methods Appl. Mech. Eng. **38**, 185 (1983).