WAVELETS: CHALLENGES FOR FLUIDS ENGINEERING

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ABSTRACT

We first reviewed the new progress of wavelet analysis in experimental fluid mechanics. As multiscale identification techniques, we then presented two new applications of wavelets in experimental fluid mechanics. Finally, we state several perspectives and point out where new methods need to be developed in order to improve our understanding of turbulent structure.

Key Words: Multiresolution Analysis, Multi-Scale Turbulent Structure, Wavelet Transform

1. INTRODUCTION

The early works on wavelets were in the 1980's by Morlet, Grossmann, Meyer, Daubechies, Mallat, and others. Now, wavelets have become pervasive in several diverse areas such as mathematics, physics, digital signal processing, image processing, computer graphics, geophysics, astrophysics, fluid mechanics, biomedical engineering, medicine, and others. New tools are available for efficient data compression, image analysis, and signal processing, and there is a great deal of activity in developing wavelet methods for use in these fields. The application of the wavelets appearing in the area of fluid mechanics started from 1988. Until now, numerous papers on this topic have been published rapidly, and succeeded in demonstrating the rich potentials. These researches can be roughly divided into two groups: (1) identifying the turbulent or eddy structure by the wavelet analysis of experimental data and simulation data; (2) developing turbulence modeling and numerical methods using wavelet bases. It is impossible to write a review about all topics. In this paper we focus on how to extract the multi-scale turbulent structures based on wavelet analysis of the experimental data. Many reports on application of wavelet analysis in experimental fluid mechanics have been summarized by Li [(1)]. Here we only describe some literature published from 1998 at the time of submission.

We first review the applications of continuous wavelet transform. In order to identify coherent structure of turbulent shear flow, Li [(1)] developed wavelet correlation method based on the continuous wavelet transform. For detecting Reynolds stress reversals phenomenon in frequency space, Li [(2)] also proposed several new definitions of the wavelet-based statistical turbulent quantities. The secondary flow structures of a turbulent bounded jet in both Fourier and physical spaces were revealed based on the wavelet cross spectrum function by Li et al. [(3)]. Recently, Bonnet et al. [(4)] summarized the one-dimensional continuous wavelet transform as one of coherent stricture education methods in their study.

On the other hand, the researches on applications of discrete wavelet transform were also active. Staszewski et al. [(5)] analyzed coherent structures in the wind fluctuations using the discrete wavelet transform. Li et al. [(6)] developed a new procedure to evaluating coherent structures in the dimension of time and scale based on the discrete wavelet transform of velocity signals. Li et al. [(7)] also successfully extracted the multiresolution turbulent structures using the turbulent image analysis of the two-dimensional orthogonal wavelet transform.

Recently, there is a new application in the image processing of fluid measurement. Li et al. [(8)] developed an application of wavelet image compression technique to PIV for improving spatial resolution and reliability,

Our paper is organized as follows. We first present two main applications of wavelets in experimental fluid mechanics as multiscale techniques. Finally, we state several perspectives and point out where new methods need to be developed in order to improve our understanding of turbulent structure.

2. MULTIRESOLUTION ANALYSIS OF VELOCITY VECTORS

It is well known fact that the turbulent near-wake of a circular cylinder has a high degree of organization and

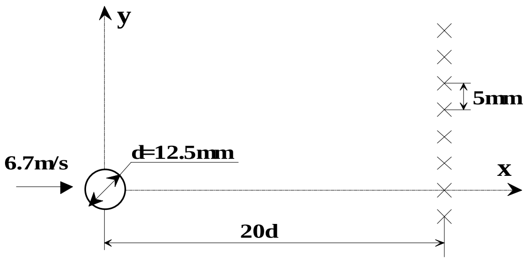

Figure 1 Experimental arrangement

is dominated by spanwise vorticity. It is therefore attractive for the purpose of identifying turbulent organized structures and clarifying their contribution to turbulent transfer process. Most existing techniques, e.g. the vorticity-based scheme, and the scheme based on critical points, focused on large-scale structures, i.e. Karman vortices. It is difficult for these techniques to deduce intermediate-scale structures. This results in a lack of experimental data of the intermediate-scale structures; our knowledge is incomplete of turbulent structures of various scales. This subsequently affects our understanding of dynamics of turbulence, for intermediate-scale structures such as rib structures play an important role in turbulent transfer process. In this study, orthogonal wavelet transform has been used to analyze the velocity data of the turbulent near-wake of a circular cylinder. The purpose of investigation was to visualize the turbulent structures of various scales and provide both qualitative and quantitative information on the intermediate-scale and relatively small-scale structures in the near-wake of a circular cylinder.

Experiments were carried out in an open return low turbulence wind tunnel with a 2.4 m long working section (0.35 m x 0.35 m). A circular cylinder (d =12.5 mm) was installed in the mid-plane and spanned the full width of the working section. Measurements were made at x/d=20 (x is the streamwise distance downstream of the cylinder) and Re_d ($\equiv U_\infty d/\nu$, where U_∞ is the free stream velocity and ν the kinematic viscosity) =5600. Using two orthogonal arrays, each comprising eight X-wires (Figure 1), velocity fluctuations u, v in the (x, y)-plane and u, w in the (x, z)-plane were obtained simultaneously with a sampling frequency 3.5 kHz. The nominal spacing between X-wires was about 5 mm. The duration of signals was about 38 s. More details of the data have been given in Zhou & Antonia [9].

The bottom plate of Fig. 2 presents the measured velocity vectors in the (x, y)-plane within the range $0 \le x/d \le 10$ and $-0.2 \le y/d \le -2.4$, where x/d=-tU_c/d (U_c=0.875 U_∞ is the convection velocity of large-scale structures) and y/d are the normalized abscissa and ordinate scales, respectively. Four vortex-like structures can be seen.

The velocity vectors were decomposed using discrete wavelet transform with Daubechies' orthogonal wavelet bases of N =20. The decomposed components of the velocity vectors are given in plate from the top to the third for wavelets levels of 4, 6 and 7 which correspond to a central frequency of f =109, 437 and

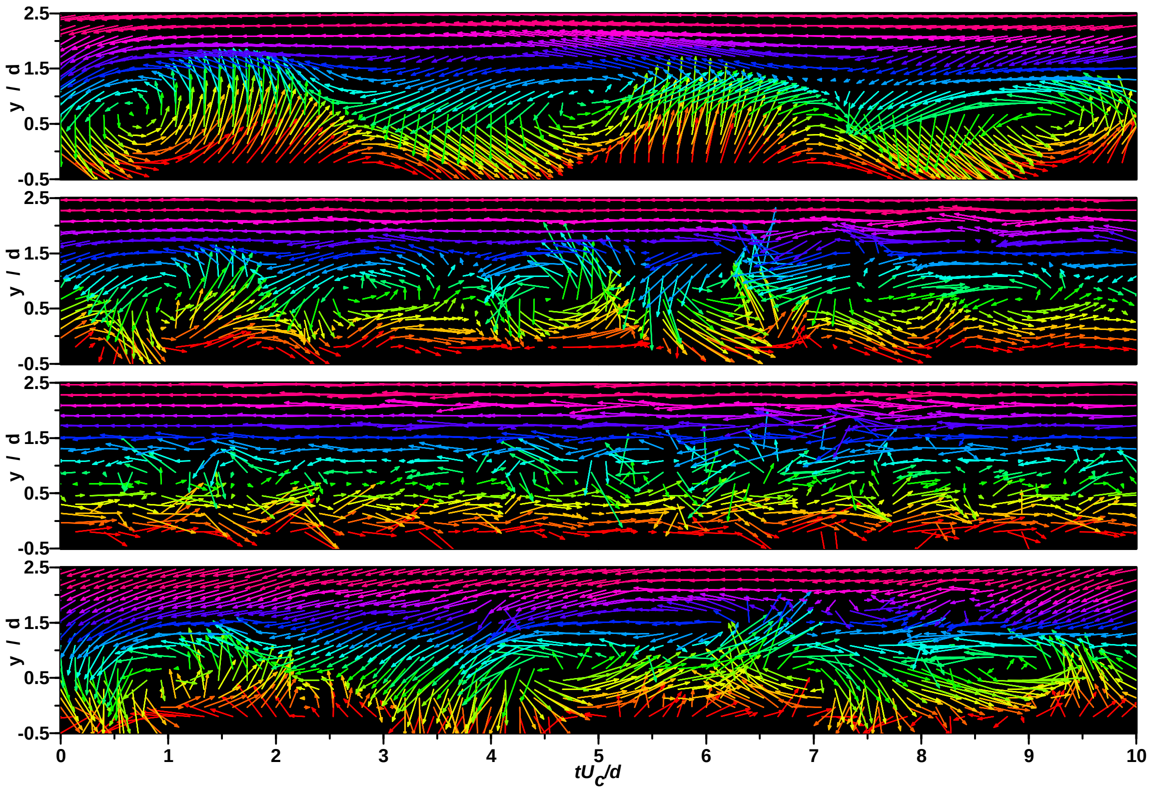

Figure 2 Multiresolution velocity vectors of wake flow

874 Hz, respectively. Three large-scale structures can be clearly observed in the top plate. They are the uppermost and energy-containing structures, i.e. Karman vortices, and correspond to the vortices appearing in the measured velocity vectors. In the second plate several vortex-like structures of intermediate scales are clearly identifiable. These structures gain energy from the large-scale structures and then pass it on to relatively small-scale structures. As the wavelet level is increased to 7 (the third plate), a number of smaller-scale structures are identifiable. These small structures seem to be unsteady and have different strength. Note that it is difficult to identify the intermediate-scale and relatively small-scale structures in the measured velocity vectors (the bottom plate). Although preliminary, the results presented here demonstrate that the orthogonal wavelet transform technique can be used effectively for decomposition and analysis of multi-scale turbulent structures.

3. MULTIRESOLUTION ANALYSIS OF TURBULENT IMAGE

Mallat and Meyer formulated the theory of multiresolution analysis in the fall of 1986, in order to provide a natural framework for the understanding and construction of wavelet bases. The goal of the multiresolution analysis is to get a representation of a function that is written in a parsimonious manner as a sum of its essential components. That is, a parsimonious representation of a function will preserve the interesting features of the original function, but will express the function in terms of a relatively small set of coefficients. Thus overcoming limitations of the two-dimensional continuous wavelet transform that cannot reconstruct the original function. In this paper, we focus on multiresolution analysis of multi-scale turbulent structure. It is well known that an image often includes too much information for real time vision processing. Multiresolution algorithm process less image data by selecting the relevant details that are necessary to perform a particular recognition task. Coarse to fine searches processes first a low-resolution image and zoom selectively into finer scales information. Multiresolution analysis is the result of a two-step process. Images are first decomposed into wavelet components and their wavelet spectrums are obtained. Reconstruction or inverse discrete wavelet transform is then done at each scale, and image components are obtained in wavelet space. Although there are several families of orthonormal wavelet basis that construct the wavelet space, we use the Daubechies family with index N=20 that is not only orthonormal, but also have smoothness and compact support.

Experiments were carried out in liquid-phase turbulent-jet flows, and images of slices, which relied on laser-induced fluorescence digital-imaging techniques, through the three-dimensional scalar filed of round momentum-driven turbulent jets were obtained. Transverse sections in the far field of the jet, at downstream position z/d=275 (jet-nozzle diameter d is 2.54mm), were recorded on a cryogenically cooled

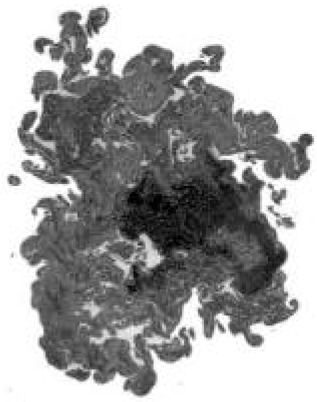

Figure 3 Original image of a turbulent jet

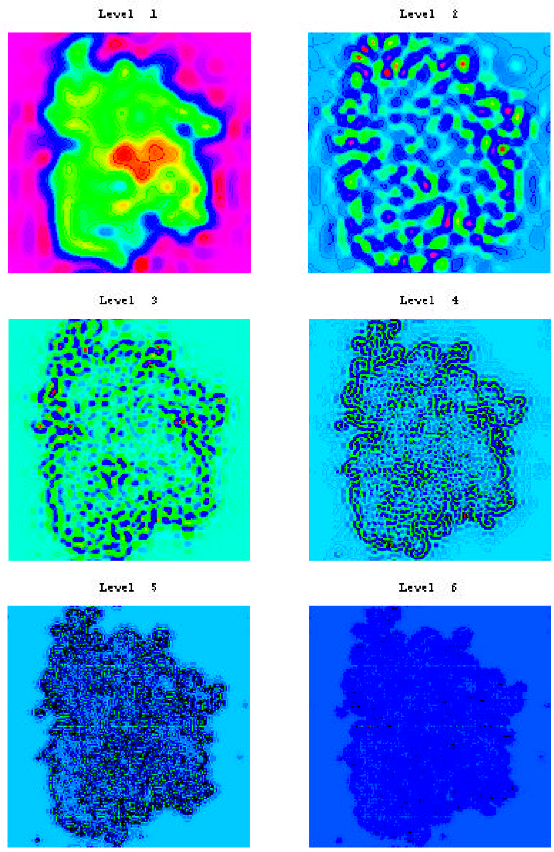

Figure 4 Multiresolution images of a turbulent jet

1024x1024 pixels CCD camera. The measured image of the jet-fluid concentration with Re=4.5x10^3 at downstream position z/d=275 is shown in Fig.3. More details have been given in Catrakis and Dimotakis [(10)].

This original image is decomposed into six wavelet spaces based on the two-dimensional orthonormal wavelet transform, and results of image components with six levels are shown in Fig.4. The top column, going from the second image of left to right, corresponds to levels 1, 2 and 3. The bottom column, going from left to right, corresponds to levels 4, 5 and 6. The sum of six image components can reconstruct the original image.

In Fig.4, false-colors have been assigned to the scalar values of wavelet transform, and the highest concentration is displayed as a deep red and the lowest as purple. Blue in each signifies the zero value. These images provide further evident of multi-scale structures in turbulent jet and may easily extract important scales that dominate the flow structure. From the image for the lowest level 1, which corresponds to the broader scale of *a*=43.0~107.5*mm*, the interior of the flow may be identified by the blue line. A large peak that contains three peaks can be clearly observed near the center of jet. By comparing the original image, these peaks imply that a large-scale structure consists of three vortices. They are the uppermost and energy-containing vortices. With the broader scale of *a*=14.3~43.0*mm*, as shown in the image of the level 2, a lot of stronger peaks mainly appear in the edge of flow region, and correspond to the positions of vortices at this scale range. These vortices are more active in the shear layer and dominate the turbulent mixing process, which are referred to as the coherent structure of the problem. These vortices gain the energy from the large-scale vortices and then transit it to the smaller-scale vortices. As decreasing scale to *a*=7.2~14.3*mm* at levels 3, peaks mainly concentrate on islands or lakes (as described in Catrakis and Dimotakis [(10)]) of the flow region edge. The distribution of peaks indicates that vortices also undertake the turbulent mixing process within this scale range in this region. When the broader scale reaching to *a*=3.6~7.2*mm*, as shown in the image of the level 4, edges of vortex within this scale range can be clearly observed. As increasing resolution to *a*=1.8~3.6*mm*, a finer approximation of the original image can be obtained at level 5. A clear distribution of vortex edges with smaller-scale can be observed, which is the "zoom-in" the image of the level 4. That is an important feature of multiresolution analysis. The image of level 6 describes the distribution of fine streamlines with the broader scale of *a*=0.8~1.8*mm*. Edges of the smallest-scale vortex in this problem can be observed everywhere in the interior of the flow. This means that the smallest-scale vortices exist in the whole flow field. From above results, it is can say that the edges of the vortices at different resolutions or scales and the coherent structure may be easily extracted by wavelet multiresolution analysis.

To sum up the major characterististics of flow structure, three types of flow structure, which are the large-scale structure near the center of jet, coherent structure in the shear layer and the small-scale vortices in the whole flow field, are of most significance and dominate the turbulent structure in jet.

4. CONCLUSION

The important feature of wavelet technique is to extract the instantaneous information from fluid field. We believe that the wavelets may become standard tool for the identification of multi-scale turbulent flow. To achieve this aim, one of important work is develop a better interpretation of wavelet results, i.e., we *should* associate wavelet results with turbulent structure.

In the near future one of challenging work is to characterize the three-dimensional flow structure by utilizing wavelets with the development of flow measurement technology.

As an application in the image processing of fluid measurement technology, wavelets will also be a promising technique.

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