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DATA HANDLING METHODOLOGY FOR DISCRETE WAVELETS AND ITS APPLICATIONS TO THE DYNAMIC VECTOR FIELDS

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A discrete wavelet transform is one of the effective methodologies for compressing the image data and extracting the major characteristics from various data, but it always requires a number of target data composed of a power of 2. To overcome this difficulty without losing any original data information, we propose here a novel approach based on the Fourier transform. The key idea is simple but effective because it keeps all of the frequency components comprising the target data exactly. The raw data is firstly transformed to the Fourier coefficients by Fourier transform. Then, the inverse Fourier transform makes it possible to the number of data comprising a power of 2. We have applied this interpolation for the wind vector image data, and we have tried to compress the data by the multiresolution analysis by using the three-dimensional discrete wavelet transform. Several examples demonstrate the usefulness of our new method to work out the graphical communication tools.

Keywords: Wavelets transform; Fourier transform; interpolation; multiresolution.

AMS Subject Classification: 22E46, 53C35, 57S20

1. Introduction

Major applications of the wavelet transform are both wave form analysis and image data compression.[1] One of the distinguished properties of the wavelet transform is extraction of major dominant factors from raw data. This method is available to analyze scalar data as a noise reduction method. In addition, the multiresolution analysis by the wavelets transform is suitable to compress data for transmission of the signal by removing random noise. However, one of the problem of the discrete wavelet transform is a restriction that the number of data should be in the power of 2.[2] Previously, we used to cut the number of data or otherwise add zero elements.[3,4] Therefore, part of the data obtained by field observations or laboratory experiments are not frequently applied for the wavelet transform. The cutting of the number of data means the lost the characteristic data, and the adding of zero elements means changing the characteristic data.

Any raw data can be represented in terms of the Fourier series, then the Fourier transform method proposed in this paper makes it possible to arrange the number of data in keeping frequency characteristics of the raw data for the discrete wavelets analysis. To demonstrate the usefulness of our methodology, we apply our method to the wind vector image data. As a result, it is verified that our method never lose the characteristics of original data on applying three-dimensional discrete wavelets transform.

2. Interpolation of One-Dimensional Periodic Data

In this chapter, we apply our Fourier transform method to a simple one-dimensional data in order to verify the validity of our approach.

2.1. *Sample data*

At first, we define a periodic function expressed as follows:

$$f(t) = \cos(2t) - \cos\left(3t + \frac{\pi}{4}\right) + \cos\left(6t - \frac{\pi}{6}\right) + \sin(t) - \sin\left(4t - \frac{\pi}{8}\right) + \sin(18t). \tag{2.1}$$

The discrete sampling data are obtained from Eq. (2.1) at an interval of $2\pi/n$, i.e. $t = 0, 2\pi/n, \ldots, 2\pi(n-1)/n$ and $n = 128$, and shown in Fig. 1.

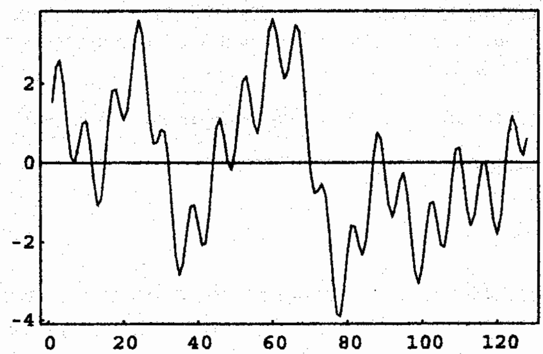


Fig. 1. Sample data calculated by Eq. (2.1), ($t = 0, 2\pi/n, \ldots, 2\pi(n-1)/n, n = 128$).

2.2. *Fourier transform of sample data*

We apply a Fourier transform of the sample data shown in Fig. 1. Figure 2 shows the Fourier spectra of the sample data. Fourier spectrum reveals the peaks at frequency of $2, 3, 4, 6$ in cosine waves, and at frequency of $1, 3, 4, 6, 18$ in sine waves, as expected.

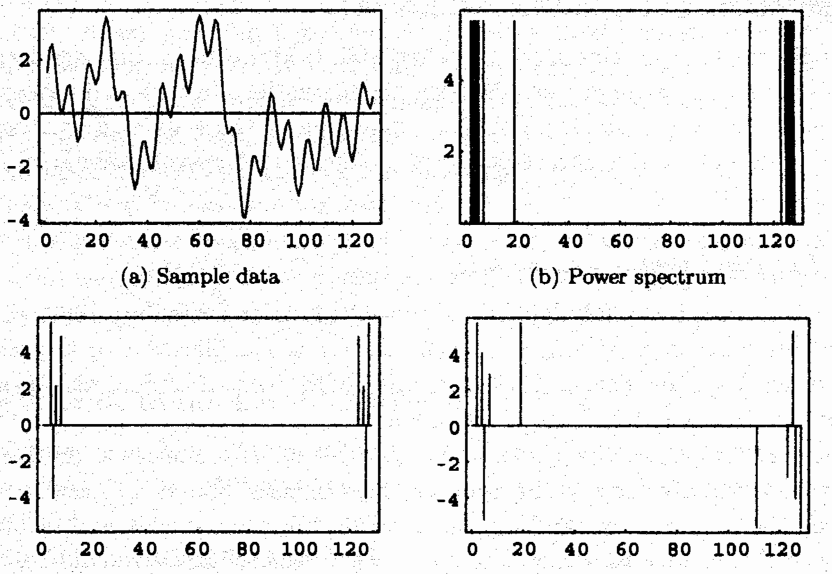
2.3. *Interpolation of the sample data*

In general, a function, $f(t)$, can be expanded in discrete real Fourier series as follows:

$$f(t) = \frac{1}{\sqrt{n}}\left(\frac{a_0}{2} + \sum_{r=1}^{n/2} a_r \cos\left(\frac{2\pi rt}{n}\right) + \sum_{r=1}^{n/2} b_r \sin\left(\frac{2\pi rt}{n}\right)\right), \tag{2.2}$$

where n is the number of discrete data. The value at arbitrary time, $t = t$, is obtained as the sum of sine and cosine waves in Eq. (2.2). When the discrete data are interpolated for satisfying the number of data of a power of 2 by using the Fourier transform technique, the new data set has also the same Fourier coefficients as the original discrete data before the interpolation.

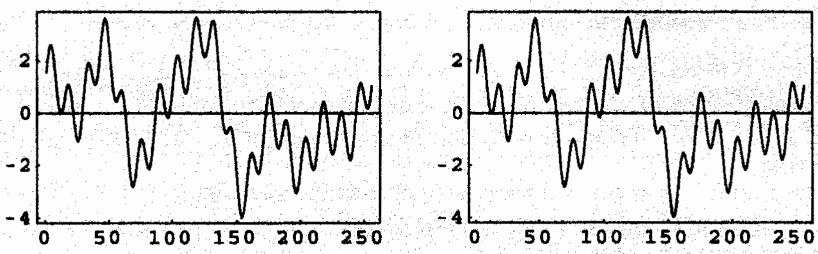
Sample data can be expanded in time series shown in Fig. 2 and values of the function, $f(t)$, are calculated for any time, $t = t$. The sample data can be increased from 128 to 256 by using this real fourier series. Figure 3 shows a comparison of the interpolated data with calculated data by Eq. (2.1) for $n = 256$.



(a) Sample data (b) Power spectrum

(c) Real parts (d) Image parts

Fig. 2. Sample data and its Fourier spectrum. (a) Sample data in Fig. 1; (b) Power spectrum of Fourier coefficients; (c) Real parts of Fourier coefficients; (d) Imaginary parts of Fourier coefficients.



(a) Interpolated data (b) Sample equation

Fig. 3. Comparison of the interpolated sample data with calculated data. (a) Interpolated data (data number of 256); (b) Calculated data by Eq. (2.1), $(t = 0, 2\pi/n, \ldots, 2\pi(n-1)/n, n = 256)$.

We evaluate a correlation coefficient between the interpolated data by Fourier transform and the original sample data calculated by Eq. (2.1), in order to investigate that the calculated data resembles the original sample data. In this case, the correlation coefficient is 1.0, so that our interpolating method never change the nature of original data. Namely, the increase of the interpolated data into the time series of digital data by the above method does not change the Fourier coefficients. If the linear interpolation is applied to the time series of digital data, then Fourier coefficients are naturally different from the coefficients before the interpolation. Therefore, the interpolation method using the Fourier Transform is reasonable methodology to one-dimensional data.

3. Interpolation of Two-Dimensional Data

We apply our interpolation method to the wind field for meteorological data. The wind is mainly constituted from the periodic fluctuations in time and space, therefore these data will be suitable to the application of the wavelet transform. The typical period motions in atmosphere are one day, about a week, annual and interannual periods.

3.1. *The wind vector image data*

Figure 4 shows the monthly mean winds at 250 hPa surface on July 1993. The horizontal distribution of the wind is available for the application of the wavelets transform to vector fields. The data are NCEP/NCAR reanalysis ones with $2.5° \times 2.5°$ grid size (longitude $\times$ latitude).

Also, Fig. 4 shows the global wind distribution of summer in the Northern Hemisphere and of winter in the Southern Hemisphere. The mean speed of the jet stream is about 30 m/s in the Northern Hemisphere, while it is 40 to 50 m/s in the Southern Hemisphere in winter. The mean wind velocity at the middle latitude in the latter dominates over one of the former.

From July to August of 1993, the atmospheric temperature in Japan was abnormally low in comparison with the mean one for 30 years. The characteristics of the

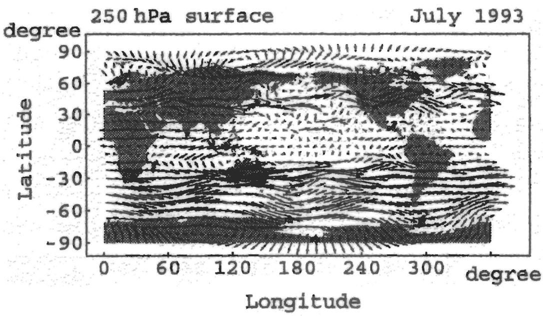


Fig. 4. Monthly mean wind data on July 1993.

eastward strong wind around the Japan, i.e. the jet stream, located south (37° to 47°N) in the summer of 1993.

The number of data is 73×144. So, our previous investigated region was not the whole earth (the number of data is 64×128), but was limited from 75°S to 82.5°N and 0°E to 42.5°W.[4,5] However, we can interpolate in the wind data by using our Fourier transform method, and this leads to the studying of the region as the whole earth.

3.2. *Interpolation of the wind data by the Fourier transform*

The number of original wind data obtained by the observations was 74×144. Then, the number of data is possible to be rearranged to 64×128 for the wavelet analysis. The Fourier spectrum in the two-dimensional field is firstly calculated at each column and line from the original wind data. The Fourier coefficients at each column and line are used to interpolate the discrete data in the wind field. As the wind is a velocity vector, the above method is applied to both north-south and east-west components of the wind data for the interpolation. Figure 5 shows the interpolated wind data by our Fourier transform method.

Thus, the Fourier coefficients of the interpolated wind data shown in Fig. 5, which are obtained by the two-dimensional Fourier transform, agree with the ones of the original wind data in Fig. 4, except for the high frequency.

4. Application to the Two-Dimensional Wavelets

The wavelets transform is applied to the interpolated wind data shown in Fig. 5. In this study, the wavelets spectra of wind are calculated under the 20th-order Daubechies base function in north-south component and the 16th-order Daubechies base function in east-west component.[5]

The inverse wavelets transform also gives the distribution of wind velocity, but the resolution of the wind field is possible to be changed by the number of data. We try to show the results of different resolution in Figs. 6 and 7.

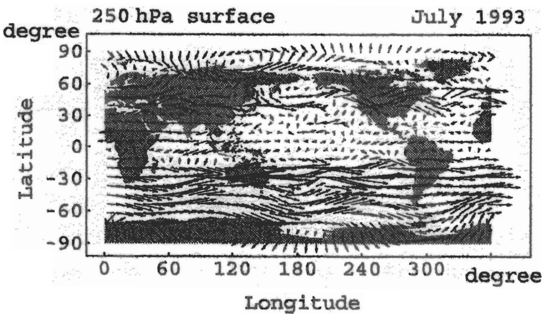


Fig. 5. Interpolated monthly mean wind data on July 1993. The number of data is 64 × 128.

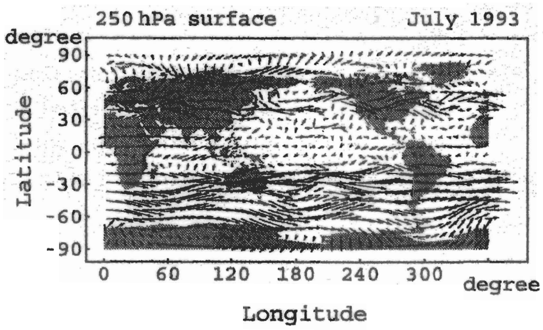


Fig. 6. Wind vectors recovered from the top 32 × 64 region shown in Fig. 5.

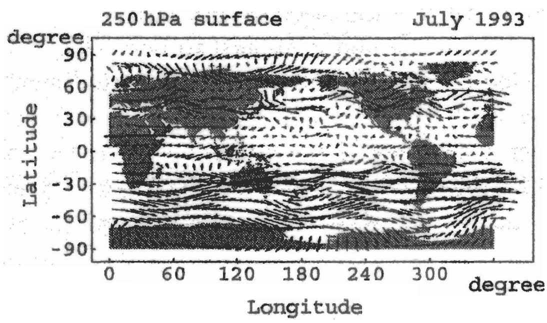


Fig. 7. Wind vectors recovered from the top 16 × 32 region shown in Fig. 5.

Figures 6 and 7 show a half of level and a quarter of level, respectively, in both north-south and east-west components of the wind velocity. The resolution of a half of level implies a reproduction by using the number of data of $1/2 \times 1/2$ of the original interpolated data in the inverse wavelet transform, that is, the data compression is made for the transform. The resolution of a quarter of level is, naturally, the data number of $1/4 \times 1/4$ of the original one.

The magnitudes of omitted high frequency vector data by compression are about one-fifth smaller than those of the vector shown in Figs. 6 and 7. Saito (1998) employed a correlation coefficient between the recovered and raw data in order to confirm the recoverability of data.[1]

As the wind is a velocity vector, we will apply a matrix correlation ρ and define recovery ratio as follows,

$$\rho = \frac{\langle \boldsymbol{X} \cdot \boldsymbol{Y} \rangle}{\|\boldsymbol{X}\| \cdot \|\boldsymbol{Y}\|}, \tag{4.3}$$

where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are matrices. The matrix correlation calculated by Eq. (4.3) (recovery ratio) is 0.999 in Fig. 6 and 0.993 in Fig. 7. Both results show good recovery.

5. Application of the Three-Dimensional Wavelets

In Sec. 4, the monthly mean data of global wind, i.e. on July 1993, are applied to the two-dimensional wavelet transform. In this section, we will include the time variations of the global wind, and treat them as a three-dimensional data. We apply the three-dimensional wavelets to wind image data, and calculate the compressed wind image data by means of the multiresolution analysis.

5.1. *Method of the three-dimensional vector wavelets*

When transpose l by m by n, three-dimensional matrix, $\boldsymbol{M}$, is written as

$$[\boldsymbol{M}_{lmn}]^T = \boldsymbol{M}_{mnl}, \tag{5.4}$$

the wavelet transform of three-dimensional matrix, $\boldsymbol{M}$, is carried out by

$$\boldsymbol{S} = [\boldsymbol{W}_n \cdot [\boldsymbol{W}_m \cdot [\boldsymbol{W}_l \cdot \boldsymbol{M}_{lmn}]^T]^T]^T, \tag{5.5}$$

where $\boldsymbol{S}, \boldsymbol{M}, \boldsymbol{W}_n, \boldsymbol{W}_m$ and $\boldsymbol{W}_l$ are the l by m by n wavelet, l by m by n original, m by m, n by n and l by l wavelet transformation matrices, respectively. When the element of matrix, $\boldsymbol{M}$, is composed of the two-dimensional vector

$$\boldsymbol{M} = [U + V], \tag{5.6}$$

the wavelet transform spectrum is carried out by two independent wavelet transforms, i.e.

$$\boldsymbol{S} = [\boldsymbol{W}_n \cdot [\boldsymbol{W}_m \cdot [\boldsymbol{W}_l \cdot U_{lmn}]^T]^T]^T + [\boldsymbol{W}_n \cdot [\boldsymbol{W}_m \cdot [\boldsymbol{W}_l \cdot V_{lmn}]^T]^T]^T. \tag{5.7}$$

5.2. *Animation wind vector image*

The sequential global wind data for 64 months from March 1992 to October 1994 are regarded as one of the animation wind vector image data. The number of data is $64 \times 64 \times 128$.

The wavelet spectra of the wind was calculated under the 8th-order Daubechies base function in time axis direction, the 20th-order Daubechies base function in south-north direction and the 16th-order Daubechies base function in east-west direction.[5]

The multiresolution analysis gives a time axis direction, east-west direction and south-north direction. In addition, it prepares the results of the resolution of level 1/2 and level 1/4 of the interpolated original data. The resolution of level 1/2 means for the wind field to be reproduced by the data number of $1/2 \times 1/2 \times 1/2$ of the interpolated original data by the inverse wavelet transform. The resolution of level 1/4 is the same as the data number of $1/4 \times 1/4 \times 1/4$ of them.

The reproduced three-dimensional animation-image is obtained by the same size ($64 \times 64 \times 128$) as the original ones. We cannot show the animation of the three-dimensional wind, so the result of July 1993 is shown in Figs. 8 and 9.

Figures 8 and 9 show the vector wind data recovered from the $32 \times 32 \times 64$ region and the $16 \times 16 \times 32$ region, respectively.

The magnitudes of omitted high frequency vector data by compression are about one-third smaller than those of the vector shown in Figs. 8 and 9.

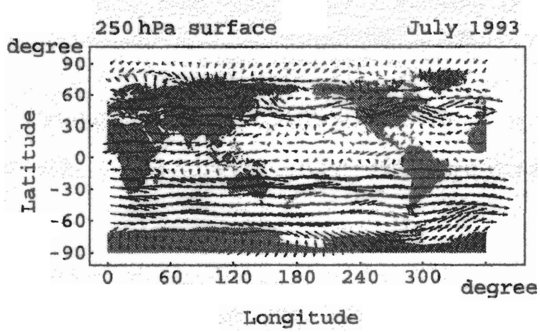


Fig. 8. Wind vectors recovered from the top $32 \times 32 \times 64$ region shown on July 1993.

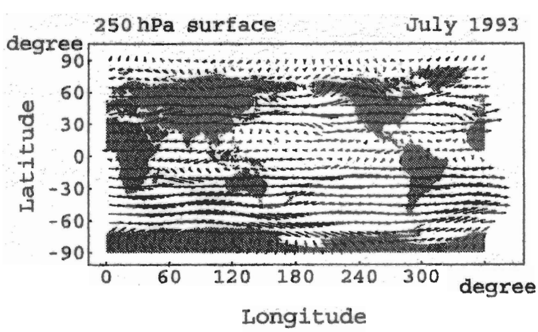


Fig. 9. Wind vectors recovered from the top $16 \times 16 \times 32$ region shown on July 1993.

The matrix correlation calculated by Eq. (4.3), i.e. recovery ratios are 0.984 in Fig. 8 and 0.967 in Fig. 9, which are very high. Figures 8 and 9 show the wind vector after compression and removal of the noise data. Its distribution is very similar to the raw data shown in Fig. 5, and these are the same characteristics in July 1993.

6. Summary

A number of data for a discrete wavelet transform is required to be a power of 2, and therefore, part of the data obtained by field observations or laboratory experiments are not frequently applied for the analyses. In this paper, we have shown that the Fourier transform is a useful methodology to interpolate and extrapolate the data for increasing or decreasing the number of data for the discrete wavelet analysis. We have applied this method for the wind vector image data, and we have tried to compress the data by the multiresolution analysis by the three-dimensional discrete wavelet transform. Several examples have demonstrated the usefulness of our new method to work out the graphical communication tools.

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